

The principles demonstrated in the articles presented so far can be used in the analysis of rotating objects. The study of rotating objects presents additional challenges that were not present in the study of objects traveling with linear motion. The simplest rotating object is a linear bar spinning about its center. However, this is not an interesting case due to the simplicity of the experiment. The interesting cases are those that involve objects with circular configurations.

THE ROTATING HOOP

The simplest circular object is the hoop, a short, hollow cylinder with a thin wall section. The hoop rotates around an axis passing through the geometric radius of the hoop. This axis is perpendicular to the plane of the hoop (the hoop lies flat against this plane). See Figure 1, where a typical hoop is shown in the plane of the page.

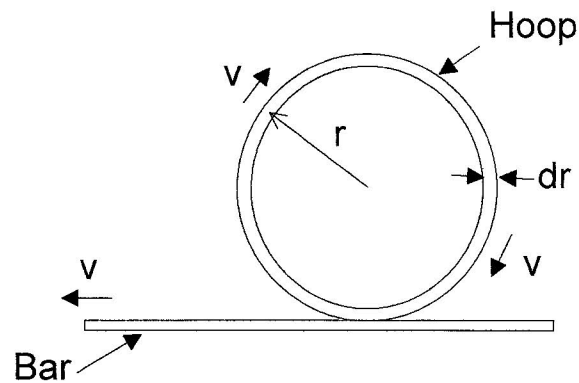


Figure 1

The hoop is described by a radius r which locates any point on the hoop from a fixed center position. The thickness of the hoop is described by the infinitesimal distance dr . The rotation of the hoop in the plane of the hoop around its geometric center will give the hoop circumference a linear velocity v which can be expressed as the relativistic velocity $\beta = v/c$. Under the Special Relativity theory presented in other articles, the circumference of the hoop might be expected to undergo “length contraction” similar to that of a linear bar at a similar velocity. If the circumference of the hoop is C_0 when $v = 0$ and the circumference of the hoop is C when v has a value greater than zero, then:

$$C = C_0 \sqrt{1 - \beta^2} \quad (1)$$

To further examine what is happening in (1), imagine that the outside of the hoop of Figure 1 is profiled with gear teeth that are identical to those on the mating surface of the bar that is also shown in that figure. When the hoop and bar are stationary, the gear teeth on each perfectly mesh with the other. The hoop can be rolled down the bar at slow speed with all teeth meshing in succession. Now imagine that the bar is stationary and the hoop rolls along it so that the center of the hoop has a velocity v . As velocity v is increased, the gear teeth must continue to mesh as before without relativistic effects. This is certain because the portion of the hoop that touches the bar is momentarily stationary relative to the bar. Other portions of the hoop have various velocities, but the meshing gear teeth have no relative velocity at the point of contact.

Now imagine that the position of the hoop is stationary relative to the observing reference frame, but the circumference has velocity v , as shown in Figure 1. The bar also has velocity v as shown in Figure 1. The gear teeth must also mesh perfectly in this case because this is exactly the same experiment as the one just mentioned, except observed from a different reference frame (that reference frame having velocity v). The bar experiences length contraction, so the portion of the hoop that touches the bar must also experience an identical length contraction.

Of course, any location on the hoop could be touched by a bar passing with a tangential velocity v , so all portions of the hoop must simultaneously experience length contraction. The total circumference of the hoop must therefore experience length contraction, as shown in (1). This means that the diameter of the hoop must also experience a similar contraction.

ENERGY AND CIRCUMFERENTIAL CONTRACTION

If the spinning hoop gets smaller in diameter, this presents a potential problem. See Figure 2.

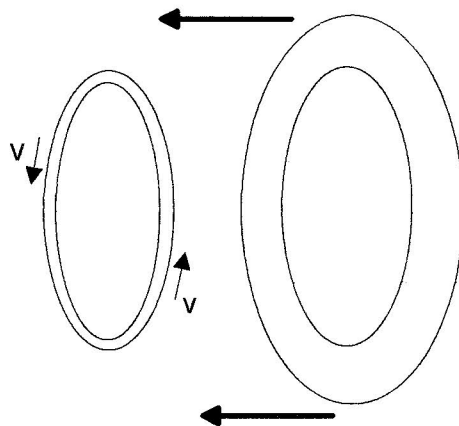


Figure 2

In Figure 2, a hoop is spinning around its axis with circumferential velocity v . It has a circumferential length contraction as given by (1). A much thicker hoop is stationary nearby. The inner diameter of this stationary hoop has a circumference that is an exact fit for the contracted circumference of the spinning hoop. Slowly, the larger hoop is maneuvered over top of the spinning hoop, as shown by the large arrows in the figure. When the two hoops are in perfect concentric fit, the spinning hoop velocity is brought to zero.

This experiment has a potential conflict with the Law of Conservation of Energy. Suppose the energy required to get the hoop spinning initially is KE . To make the experiment easier to envision, also suppose that the two hoops have mating surfaces that are frictionless. After the two hoops are in concentric position, the inner hoop velocity can be brought to zero and the experiment will recover all of KE that was expended initially to get the hoop up to speed. However, the smaller hoop is now jammed into the larger hoop, creating a stress energy (potential energy). This stress energy SE is created by the experiment. This cannot happen, so the hoop diameter must not contract. The spinning hoop is a paradox.

Now consider what the observers on the spinning hoop see. They see an identical hoop fixed in the stationary reference frame passing by at velocity v and believe that its circumference and diameter should be contracted. In linear motion experiments, this difference in stationary and moving reference frame observations is easily explained. But with spinning hoops, the linear explanation cannot apply. If the stationary observers see the spinning hoop getting smaller in diameter, then so must the observers on the hoop. However, the observers on the hoop do not see hoop length contraction. The hoop is stationary to them and does not undergo a diameter contraction. Either the spinning hoop gets smaller in diameter or it doesn't. Both sets of observers must agree on this. They both must also agree on the diameter of the stationary hoop.

Therefore, any contraction of the circumference of the spinning hoop must produce a compressive stress in the hoop that is seen by those on the hoop. There is a battle going on in the spinning hoop between two different phenomena. Special Relativity is trying to contract the hoop circumferentially and this is opposed by compressive forces generated in the hoop material. The spinning hoop does not contract according to (1). The actual diameter and circumference of the hoop in motion will be a compromise between Special Relativity effects and material compressive properties. There is a related discussion on length contraction and the linear movement of bars in the article *The Real Ladder Paradox*.

However, the length contraction of a moving bar is different because the ends of the bar are not constrained. They can go wherever Special Relativity dictates and this resolves potential conflicts. But, the circular nature of a hoop produces this conflict with the Law of Conservation of Energy, as shown in Figure 2. Therefore, any contraction of the circumference that takes place must be accompanied by a compression of the material. This compression will automatically generate SE and resolve the conflict. The two hoops of Figure 2 can still be placed concentrically, but the mating diameters will not be described by (1). As the inside hoop velocity is brought to zero, any resulting stress energy is not created, but must have been input to the spinning hoop in the beginning of the experiment during the initial hoop acceleration.

All of the theory just stated relies on assumptions of a thought experiment taking place in a perfect relativistic universe. In the real universe, a hoop spinning as described above would stretch out and break due to centripetal force long before any relativistic effects could be noticeable. For reference, the stresses imposed on a spinning disc with a hole at the center are given in (2) (from Joseph E. Shigley, "Mechanical Engineering Design," McGraw-Hill, 1963). The maximum stress in the radial direction $\sigma_{r,\max}$ occurs at a radius $(ab)^{1/2}$ and the maximum stress in the tangential (circumferential) direction $\sigma_{t,\max}$ occurs at the radius a .

$$\sigma_{r,\max} = \left(\frac{3 + \mu}{8} \right) \left(\frac{\rho \omega^2 (b - a)^2}{g} \right)$$

$$\sigma_{t,\max} = \left(\frac{3 + \mu}{4} \right) \left(\frac{\rho \omega^2}{g} \right) \left(b^2 + \frac{1 - \mu}{3 + \mu} a^2 \right)$$

b = outside radius of the disc

a = radius of center hole

μ = Poission's ratio

ρ = density of the material

g = gravitational conversion constant

ω = angular rotation speed

(2)

According to (2), circumferential velocities approaching the speed of light on any geometry of disc will result in stresses far greater than any known material can withstand. The thought experiments mentioned above require the use of a stiff hoop that is also weightless. These conditions do not exist in the real universe. Could it be that Special Relativity considerations play a part in determining material properties of real materials?

ROTATION AND TIME

Hoops differ from linear bars in that there are no beginning and end points on a hoop. The concept of 'failure of simultaneity at a distance' is specifically dependent upon the 'at a distance' specification. The geometry of the hoop automatically invalidates this concept.

Assume the hoop is initially stationary and has identical clocks lined up around the circumference. All the hoop clocks are synchronized with all the clocks in the stationary reference frame. Then the hoop is accelerated to circumferential velocity v . From the equations developed in the articles *The Acceleration Law* and *Position, Velocity, Acceleration*, the clock reading of any single clock on the hoop can be calculated precisely during the acceleration (and also after the hoop achieves constant velocity v). In fact, the clock reading of every clock on the hoop is known and they all read the same thing, as seen by observers in the stationary frame. All the clocks go through identical accelerations, so all clocks must read the same. There is no 'failure of simultaneity at a distance' on the spinning hoop. If there were, then how would one clock be selected to be the clock with the calculated time?

The observers on the hoop also see their clocks reading the same. The observers on the hoop all get together after the acceleration to velocity v and compare notes. They all started with the same clock reading before the acceleration and they all went through the same acceleration. They all must conclude that they finished the acceleration with the same clock reading. In addition, all the observers on the hoop agree that all the stationary clocks next to the hoop read the same. Again, all these clocks passed by the hoop observers with identical movement histories, so they all show the same time at any point during the rotation.

A spinning hoop is an unusual case in Special Relativity where all clocks on the stationary frame and moving frame are synchronized and neither set of observers sees 'failure of simultaneity at a distance' in either reference frame. And both sets of observers agree that the time on the moving hoop is going slower than time in the stationary reference frame. They know this because the spinning hoop is at a higher kinetic energy level relative to the stationary reference frame. As has been shown in other articles, Special Relativity effects are results of energy differences between reference frames. These two reference frames are not equivalent. One is definitely experiencing an acceleration and one is not and both sets of observers know which is which.

Observers on the moving bar of Figure 1 will also have an unusual view of the experiment. As the hoop rolls over the bar, various locations on the hoop circumference will have a variety of velocities relative to the bar. Times rates on the hoop will vary according to their velocity. As the observers on the bar watch a specific clock on the hoop, they will see that it has an accumulated time effect. It is stationary when it is in a position touching the bar, so it's rate of

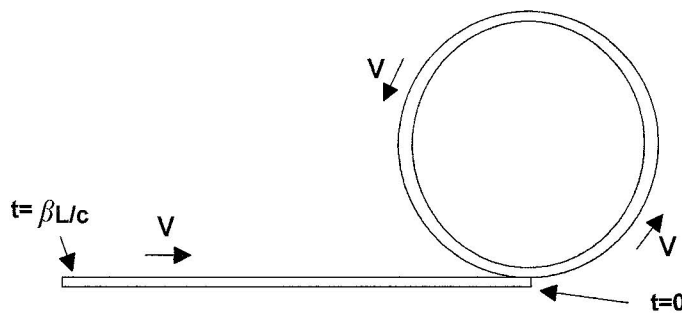


Figure 3

time passage is the same as that on the bar when it is in this position. But at all other locations on its journey, this particular clock has a velocity relative to the bar. Therefore, as the clock travels with the hoop, its clock slows down relative to clocks on the bar. When the clock is once

again at a position touching the bar, it will have a reading that is slow compared to the clock readings on the bar.

This same loss of synchronization is seen by the stationary reference frame. In Figure 3, the hoop is shown with circumferential velocity v (but stationary in position) and the bar of stationary length L is passing by with the same velocity. The length $L\sqrt{1-\beta^2}$ of the bar is exactly equal to the circumference of the moving hoop (assume the length of the bar matches the actual circumference of the hoop with the compressive effects mentioned previously). At the instant shown, the clock on the front of the bar reads zero, as does the clock next to it on the hoop. The clock on the back of the bar reads ahead of the clock on the front of the bar as shown. After one complete revolution of the hoop, the original clock on the hoop will now be next to the last clock on the bar. The amount of time shown on stationary frame clocks for one revolution of the hoop is $\frac{L\sqrt{1-\beta^2}}{\beta c}$. The clock on the hoop will now read $\frac{L(1-\beta^2)}{\beta c}$ and the clock on the back of the bar now reads $\frac{L}{\beta c}$. The stationary frame also sees the clocks on the hoop run slower than the clocks on the bar.

The geometric distortions produced by relativistic velocities of circular objects produce results that are unexpected in the Newtonian world. In the article *The Real Ladder Paradox*, this same idea was explored for linear bars. A spinning hoop forces clocks on its circumference to have the same readings (not have failure of simultaneity at a distance) just like the example in *The Real Ladder Paradox*. Just like in that example, enforced simultaneity on a single circular object also places the object in a state of stress.

SOLID DISCS

If the hoop is replaced by a solid disc of material, the analysis gets more complex. A disc can be thought of as a series of nested hoops, each with its own individual circumferential velocity. As the hoops get progressively smaller in radius, the relativistic contraction of the circumference at each radius reduces. The inner hoops of the disc therefore place an outward force on the exterior hoops, further resisting the outer hoop tendency to reduce in diameter in strict accordance with (1).

A spinning disc is a unique item in the universe because it is one object that simultaneously has various locations that are at different velocities relative to each other. There is a smooth transition from relativistic speeds to non-relativistic speeds all within the same object. This transition places the disc material in conflict with relativistic effects and adds stress energy to the disc. And it requires this extra energy to be put into the disc during acceleration (compared to the kinetic energy required only to obtain the disc velocity) as seen by those in the stationary frame observing the disc. In the relativistic universe, the highest tensile stress in the material of the disc occurs on the outer fibers of the disc radius, whereas, in the Newtonian universe, the highest tensile stress occurs at the center of the disc (as expressed by (2)).

SPINNING SHAFTS

A shaft will be defined as solid or hollow cylinder that has a significant axial length. A shaft is also a disc or hoop, except that the significant axial length implies that an axial variation of events will be noticed during the experiment. All of the examples in this article discussed so far could apply to hoops or discs with significant length, but no effect on the experiments would result from the axial length (neglecting end effects in internal stress configurations). One significant influence of extended axial length in a shaft occurs when there is a linear velocity in the direction of the axis of the shaft.

In Figure 4, two identical shafts are shown. Each shaft has identical lines along the outside surface that run parallel to the axis. Both have a velocity in the direction of the axis of the shafts, but the shaft on the right also has a rotation around the axis, the direction of which is indicated by the two vertical arrows. Both shafts will exhibit length contraction, time dilation and 'failure of simultaneity at a distance' as observed by a stationary reference frame. The shaft on the right has a 'twisted' appearance to it caused by 'failure of simultaneity at a distance.' Since the trailing end of each shaft shows events at a 'later' point in time (as seen by a stationary reference frame), the trailing end of the shaft on the right will have additional rotation to it compared to the front end of the shaft.

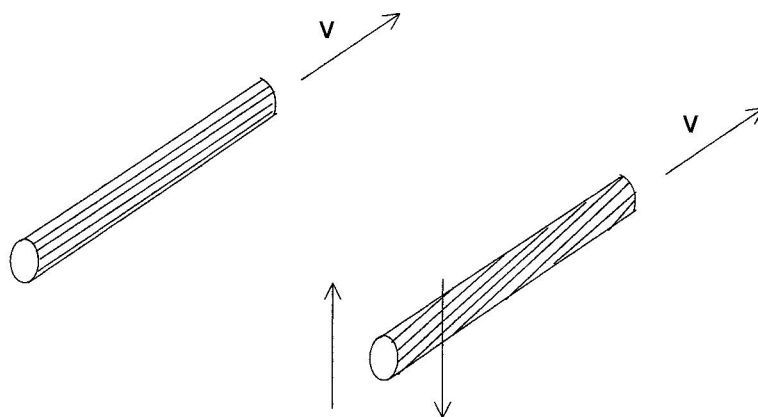


Figure 4

The shaft on the right does not feel any twisting internal stress (it is not twisted in a reference frame in which it is stationary). Similarly, shafts experiencing length contraction do not feel

compressive stress. However, the stresses already described in this article still apply and the shaft on the right has a higher velocity at the outside diameter surface due to the additional rotation velocity of that surface. Observers on the outside surfaces of each shaft will see the rest of the universe differently.

SUMMARY

The value in studying rotating objects is that this study will help to certify the results of Special Relativity that have been obtained in the more usual thought experiments involving linear object travel. If the behavior of rotating objects cannot be deciphered in a logical way using Special Relativity, then doubt is cast on the validity of Special Relativity in linear experiments because real objects do rotate.

Rotating objects such as hoops, discs and shafts are not inertial reference frames, but they can still be analyzed using the tools developed in other articles. The real conflicts with rotating objects arise because reference frames on the circumference of these objects do not have clear beginning and end points. As an observer travels circumferentially, he eventually comes back to the place where he began his journey. This is the basic conflict between Special Relativity (where reference frames are observed to have progressive effects in the direction of velocity) and the circular geometry (which cannot have progressive change in the direction of circumference). The spinning hoop is in a natural state of stress because the relativistic modification of space-time imposed by its velocity is coming in direct conflict with the natural geometry of the hoop. Phenomena depending on linear variation of properties such as length contraction and 'failure of simultaneity at a distance' cannot proceed as they did in linear experiments.

It is not possible for objects in the real world to rotate at relativistic speeds. Descriptions of the behavior of rotating objects is therefore a theoretical exercise. However, the geometry of the universe is full of circles (or near circles) and full of objects that rotate. Although theoretical, the study of circular geometry and rotation is not unimportant.

Rotating objects are unique. When a disc rotates, it is a single object that contains points traveling at relativistic speeds and points that travel at Newtonian speeds. There is a smooth transition of velocities and effects along the dimensions of this object between these two conditions. This places the object in a natural state of stress separate from the stress imposed by centrifugal force on the object material.

Rotation presents challenges to understanding Special Relativity and solving those challenges gives new information about the workings of the universe. This article presents the opportunities to ask many more questions than those that have been answered by it. The further study of rotation within Special Relativity will present challenges and opportunities beyond the achievements in linear velocity analysis.