In the article *Force and Time*, a Mysterious Force was shown to exist during a time period Δt_{AP} given by (55), but the source of this force needs to be explained. The two masses in the experiment of Figures 19 and 20 both end up with a velocity of magnitude β_X (but opposite directions) as viewed by frame B. However frame A sees one mass as stationary and the second mass having velocity β_{X2}:

$$\beta_{X2} = \left(\frac{-\beta_X - \beta_X}{1 + (-\beta_X)(-\beta_X)}\right) = \left(\frac{-2\beta_X}{1 + \beta_X^2}\right) \tag{59}$$

At velocity β_{X2}, this mass will have a momentum of P_2 as seen by frame A.

$$P_2 = \left(\frac{mc\beta_{X2}}{\sqrt{1 - \beta_{X2}^2}}\right) = \left(\frac{-2mc\beta_X}{1 - \beta_X^2}\right) \tag{60}$$

This is also the total momentum of both masses in the experiment after the experiment is over. Assume frame A sees the momentum of both masses at the start of the experiment as P_1 (the experiment begins with both masses having a frame B velocity of zero).

$$P_1 = \left(\frac{-2mc\beta_X}{\sqrt{1 - \beta_X^2}}\right) \tag{61}$$

P_2 is not equal to P_1 because the momentum seen by frame A at the start of the experiment is not complete. Frame B also starts the experiment with energy E which is used to give the masses their velocities. The momentum of this energy as seen by frame A is P_E:

$$E = 2mc^2((1 - \beta_X^2)^{-1/2} - 1)$$

$$P_E = \left(\frac{-2mc\beta_X}{\sqrt{1 - \beta_X^2}}\right)((1 - \beta_X^2)^{-1/2} - 1) \tag{62}$$

If (62) is added to (61) the momentum at the beginning of the experiment will equal the momentum at the end (60), as seen by frame A.

To evaluate the effect of the Mysterious Force on the second mass m (or on frame B) during Δt_{AP}, $-F_{XB}$ (the force on the second mass m) is multiplied by Δt_{AP} to give the momentum generated by this Mysterious Force event, P_{MF}.

$$P_{MF} = -F_{XB}\Delta t_{AP} = \left(\frac{\left(\frac{-2x_B F_{XB}\beta_X}{c}\right)}{\sqrt{1 - \beta_X^2}}\right) \tag{63}$$

The quantity $F_{XB}x_B$ is the energy used to accelerate each of the masses and is equal to the kinetic energy gain of each mass, KE.

$$KE = mc^2((1-\beta_X^2)^{-1/2} - 1)$$

$$P_{MF} = \left(\frac{-2mc\beta_X}{\sqrt{1-\beta_X^2}}\right)((1-\beta_X^2)^{-1/2} - 1) \tag{64}$$

Equation (64) is the same as (62). The momentum created by the Mysterious Force, as seen by frame A, is equal to the momentum of the initial energy that is used to accelerate the masses. In other words, the Mysterious Force event is where force $-F_{XB}$ acts during time period Δt_{AP} to add the momentum of energy E to the second mass m. The second mass m becomes the location of the material momentum with a value of P_1 in addition to the momentum of energy E.

The idea that energy has momentum in this experiment should also be apparent from the view of frame B. In general, if momentum P = mv and force is defined as F = dP/dt, then:

$$F = v\left(\frac{dm}{dt}\right) + m\left(\frac{dv}{dt}\right) \tag{65}$$

The mass in this experiment would be thought of as the mass of the material which has a value m plus the mass of the kinetic energy of this material which has a value $KE = mc^2((1-\beta_X^2)^{-1/2} - 1)$, just as in (64). This defines a total value to the mass of m_T.

$$m_T = m + m((1-\beta_X^2)^{-1/2} - 1) = \left(\frac{m}{\sqrt{1-\beta_X^2}}\right) \tag{66}$$

The force F in (65) is F_{XB}. Equation (66) is differentiated to give:

$$\left(\frac{dm_T}{dt_B}\right) = \left(\frac{m\beta_X}{(1-\beta_X^2)^{3/2}}\right)\left(\frac{d\beta_X}{dt_B}\right) \tag{67}$$

Equation (65) becomes:

$$F_{XB} = \left(\frac{mc\beta_X^2}{(1-\beta_X^2)^{3/2}}\right)\left(\frac{d\beta_X}{dt_B}\right) + m_T c\left(\frac{d\beta_X}{dt_B}\right)$$

$$F_{XB} = \left(\frac{mc}{(1-\beta_X^2)^{3/2}}\right)\left(\frac{d\beta_X}{dt_B}\right) \tag{68}$$

Equation (68) is correct according to traditional relativistic calculation of the movement of mass m under the influence of force F_{XB}. The calculations leading to (68) show that frame B sees this reaction of mass m to force F_{XB} as a combination of the acceleration of the material in the mass and the kinetic energy contained in the mass, plus the effect of the mass of the energy instantaneously flowing into m. The momentum of mass m is also considered to be a combination of the momentum of the material and kinetic energy of m.

To further explain this idea, consider what happens when a force is applied perpendicular to the direction of motion, as described by equation (32) in the article *Force and Geometry.*

$$\left(\frac{F_{XA}}{F_{YA}}\right)=\left(\frac{-\beta_{XB}\beta_{YB}}{\sqrt{1-\beta_{XB}^2}}\right) \tag{32}$$

The relationship between energy flow into frame A, dE_A/dt_A, and the force F_{YA} is:

$$F_{YA}c\beta_{YA}=\left(\frac{dE_A}{dt_A}\right) \tag{69}$$

Combining (32) and (69) with (27) gives:

$$F_{XA}=-\left(\frac{\beta_{XB}}{c}\right)\left(\frac{dE_A}{dt_A}\right) \tag{70}$$

Using the relationship $E = mc^2$, the flow of energy dE_A/dt_Ainto frame A in the experiment of Figure 10 represents a flow of mass dm_A/dt_A into frame A. Equation (70) can be rewritten:

$$F_{XA}=-c\beta_{XB}\left(\frac{dm_A}{dt_A}\right) \tag{71}$$

Equation (65) is used to predict the behavior of mass m in this experiment too. The applied x-direction force in the experiment of Figure 10 is zero (F = 0 in equation (65)). The term $v\left(\frac{dm}{dt}\right)$ will not need further development in this case. The quantity $m\left(\frac{dv}{dt}\right)$ can be described as the Newtonian reaction force of mass m (Newton's third law). This reaction force is the Mysterious Force F_{XM}.

$$F_{XM}=-v\left(\frac{dm}{dt}\right) \tag{72}$$

Equation (72) is equal to (71). At the point in time of the experiment of Figure 10, frame A will see that mass m has no x-direction momentum but does have a negative x-direction acceleration. This acceleration is caused by a force acting in the negative x-direction on mass m. This force results from the momentum change of the mass of the energy flowing into mass m. As energy flows into mass m in the y-direction, this energy has a negative x-direction momentum due to it's origin in frame B (this energy has no x-direction momentum as seen by frame B). The acceleration of this incoming energy, as it becomes part of mass m, produces the Mysterious Force that accelerates mass m in the negative x-direction.

Mysterious Forces are produced by the change in the momentum of energy as the energy transfers from one inertial reference frame to another. A key factor in this idea is (47), which expresses the constant value of applied forces regardless of reference frame velocity. All these ideas can be tested by constructing an experiment from a seemingly contradictory phenomenon.

If light beams (photons) are used to generate forces, (47) might be the source of a paradox. The momentum of a photon changes depending upon the velocity of the reference frame observing the photon, so forces generated by photons may seem contrary to (47). See Figure 21.

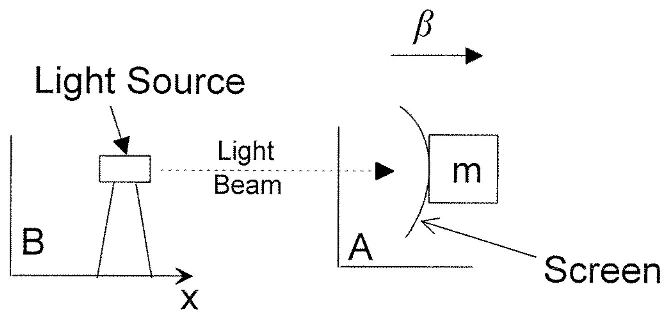

Figure 21

This experiment shows a light source located in frame B shining a beam of light to a collection screen attached to mass m in frame A. This mass is momentarily stationary in frame A and would have a velocity of β relative to frame B. There is a duplicate experiment run in the opposite direction by frame B so that the frame B momentum does not change during the experiment. This duplicate experiment is not shown.

Due to the momentum of the photons leaving the light source, there is a force generated on frame B in the negative x direction. However, the photon momentum arriving at the mass and screen stationary in frame A is not the same value as that in frame B. Photon momentum P as a function of the frequency f is given in (73).

$$P = \left(\frac{hf}{c}\right) \tag{73}$$

$$f_A = f_B\left(\frac{1-\beta}{1+\beta}\right)^{1/2}$$

$h =$ Planck's Constant

The light frequency f_A seen by frame A would be different than the frequency f_B seen by frame B. Since the forces applied by the light beams to their respective frames are related to the photon momentum within the frame, it would appear from (73) that the force observed by each frame is different.

To resolve this paradox, the acceleration method of Figure 16 will be used. Frame B begins the experiment applying no force, then applies a constant force F_B for a time period Δt. After this time period, frame B again applies no force and mass m coasts at a constant velocity. Assume a reference frame A with velocity β in the direction of F_B is observing the experiment.

The mass is stationary in frame B at the start of the experiment. Both reference frames will see the value of force that is accelerating mass m as F_B. The experiment is momentarily in the period where no force is applied. Then frame B exerts force F_B for a time period Δt. Mass m will go from velocity zero to velocity β_A in frame A and β_B in frame B. After the time period Δt is over, mass m will have momentum P_A in frame A and P_B in frame B.

$$\beta_A = \left(\frac{\beta_B - \beta}{1 - \beta_B \beta}\right)$$

$$P_A = \left(\frac{mc\beta_A}{\sqrt{1-\beta_A^2}}\right) \qquad P_B = \left(\frac{mc\beta_B}{\sqrt{1-\beta_B^2}}\right) \tag{74}$$

Inserting the expression for β_A in to the expression for P_A gives:

$$\left(\frac{P_B}{P_A}\right) = \left(\frac{\sqrt{1-\beta^2}}{1-\left(\frac{\beta}{\beta_B}\right)}\right) \tag{75}$$

If a photon had been sent by frame B to frame A, the momentum of the photon as seen by frame B would be P_{BP} and the momentum seen by frame A would be P_{AP}.

$$\left(\frac{P_{BP}}{P_{AP}}\right) = \left(\frac{hf_B/c}{hf_A/c}\right) = \left(\frac{1+\beta}{1-\beta}\right)^{1/2} \tag{76}$$

Equations (75) and (76) are equivalent if $F_B\Delta t$ is chosen to be the same as the photon momentum ($P_B = P_{BP}$). In equation (75), as mass m gets smaller, its final velocity β_B will get larger. In the limit, as mass m approaches the "mass" of a photon, then β_B would approach the value one and (75) would approach (76). The same momentum would be observed in frame A for a particle with a photon mass whether the acceleration method of Figure 16 is used to give the mass a velocity or a photon is sent directly by frame B. Photon momentum, as seen by different reference frames, is consistent with the principles expressed by (23a) and (47) where applied force is constant regardless of reference frame velocity.

For an additional evaluation of photon behavior, consider the experiment of Figure 9 in the article *Force and Geometry.* This experiment will be simulated using photons to see if

Mysterious Forces are still present without the board mechanism. For this new experiment, consider Figure 22.

This experiment shows a stationary frame B shooting photons in sequence up to moving frame A. These photons represent the flow of energy into frame A previously supplied by F_b. The photon's velocity is purely in the y-direction, so frame B once again does not feel an x-direction force during the experiment. Each photon is timed to strike frame A in the same location where they are completely absorbed. Frame A is so massive that its velocity is not significantly affected by the photons, but it can measure the impulse and energy of each as it is absorbed. The stream of photons could be treated as a continuous energy transfer. If the total momentum of all the photons in this stream is P and the total energy of all the photons is E, the force produced by this photon stream is:

$$F = \left(\frac{dP}{dt}\right) = \left(\frac{dE}{cdt}\right)$$

$$F_A = \left(\frac{dE_A}{cdt_A}\right) \tag{77a}$$

$$F_B = \left(\frac{dE_B}{cdt_B}\right) \tag{77b}$$

The total energy of the photon stream is a product of the individual energy of each photon and the total number of photons in the stream, n.

$$E_A = nhf_A \tag{78a}$$

$$E_B = nhf_B \tag{78b}$$

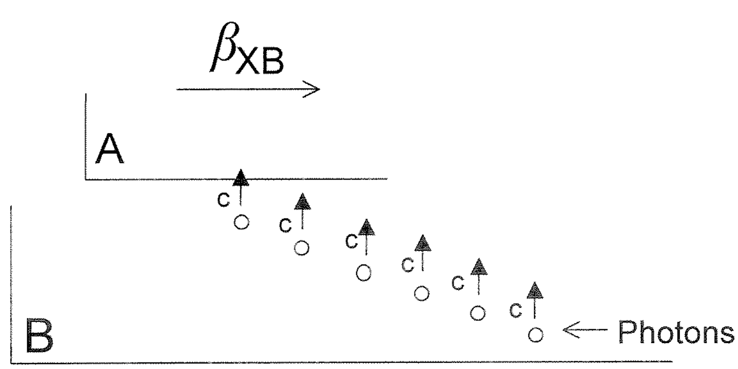

Figure 22

The photon frequency seen by frame A is higher than the frequency emitted by frame B due to time dilation. The photon rate (number of photons emitted and received per unit time) is similarly affected.

$$f_B = f_A \sqrt{1-\beta_{XB}^2} \tag{79}$$

$$\left(\frac{dn_B}{dt_B}\right) = \left(\frac{dn_A}{dt_A}\right)\sqrt{1-\beta_{XB}^2} \tag{80}$$

Combining (78) and (79):

$$E_B = E_A \sqrt{1-\beta_{XB}^2} \tag{81}$$

The energy flow rate is found from (78), (79) and (80):

$$\left(\frac{dE_B}{dt_B}\right) = \left(\frac{dE_A}{dt_A}\right)(1-\beta_{XB}^2) \tag{82}$$

Equations (77) and (82) lead to a result for the forces generated by the photon stream:

$$F_B = F_A(1-\beta_{XB}^2) \tag{83}$$

The force felt by frame A receiving the photon stream is greater than the force felt by frame B emitting the photon stream. However, the view of the experiment from frame A shows the influence of space-time geometry. See Figure 23.

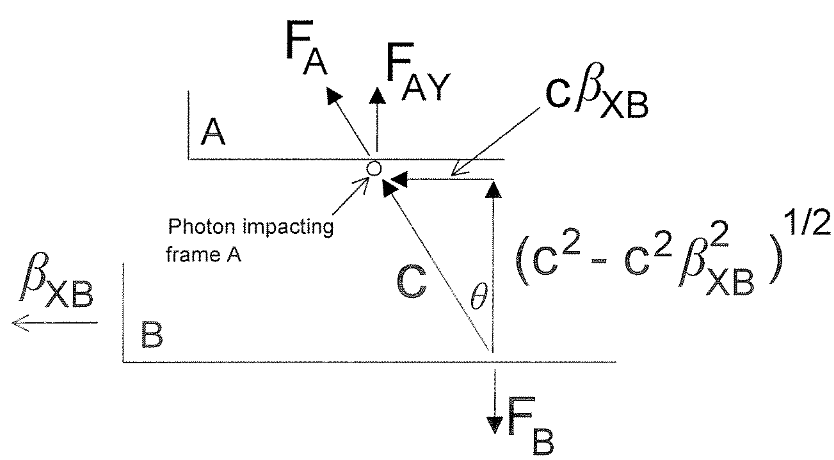

Figure 23

Note in Figure 23 that F_B is shown pointed purely in the y-direction, even though frame A will not see it this way. The force is drawn this way (as frame B would see it) to help the reader get a sense of the relationship between F_B and F_A as viewed by each reference frame.

Force F_A impacts frame A at an angle θ due to the velocity of frame A. Also, $\cos(\theta) = \sqrt{1-\beta_{XB}^2}$. Therefore, from (83) and the fact that $F_{YA} = F_A \cos(\theta)$:

$$F_B = -F_{YA}\sqrt{1-\beta_{XB}^2} \tag{84a}$$

$$F_{YA} = \sqrt{1-\beta_{XB}^2}\left(\frac{dE_A}{cdt_A}\right) \tag{84b}$$

Equation (84a) is equivalent to (23b). This experiment also produces an x-direction force in frame A which is:

$$F_{XA} = -F_A \sin(\theta) = F_A \beta_{XB} \tag{85a}$$

$$F_{XA} = -\beta_{XB}\left(\frac{dE_A}{cdt_A}\right) \tag{85b}$$

The minus sign indicates that the velocity β_{XB} of frame B as seen by frame A is in the negative direction. The ratio of the x-direction and y-direction forces as seen by frame A is:

$$\left(\frac{F_{XA}}{F_{YA}}\right) = \left(\frac{-\beta_{XB}}{\sqrt{1-\beta_{XB}^2}}\right) \tag{86}$$

This equation is equivalent to (32) or (33), except that $\beta_{YB} = 1$, due to the fact that light is now the mechanism of energy transfer.

In this case, the effect of space-time geometry on the path of the original photon velocity can be seen. The Mysterious Force F_{XA} is the x-direction component of the flow of photon momentum into frame A. Photon flow between inertial reference frames is another way that energy flows between these frames. Photon flow is easier to visualize than energy flow from a force adding kinetic energy to a mass (and photon momentum is easier to visualize than kinetic energy momentum).

This series of articles (*Position, Velocity, Acceleration*, *Force and Geometry*, *Force and Time*, and *Energy Dynamics*) shows a convergence of ideas that all lead to the same conclusion. Space-time geometry distortions are associated with the existence of forces in dynamic experiments. These forces are the result of the momentum change of energy in the experiments. This applies when using purely geometric deductions involving position, velocity and acceleration. It also applies when applying the Law of Conservation of Momentum (or Newton's Laws). And the behavior of photons in dynamic experiments gives the same result. Energy has mass and momentum which follow the laws of physics in the same ways as material mass and momentum. The change in momentum of energy produces Mysterious Forces.

As pointed out previously, inertial mass appears to have the same value as gravitational mass and gravitational fields are also associated with distortions of space-time geometry. In the study of gravity, it may be of interest to determine the relationship between gravitational forces and Mysterious Forces.