In the article *Force and Geometry*, the result (23a) gives the transformation for a force applied in the direction of the movement of an inertial reference frame.

$$F_{XB} = F_{XA} \tag{23a}$$

Equation (23a) has been shown to be true only for the exact instant where there is no x-direction velocity to mass m in frame A. The case where there is a frame A x-direction velocity to mass m is worthy of investigation.

In Figure 14, the experiment is similar to the experiment of Figure 8 except that there is no y-direction force on mass m. Frame B has an observer that is applying a constant force F_{XB} to mass m which is moving with velocity β_{XA} relative to frame A. Frame A has velocity β_X.

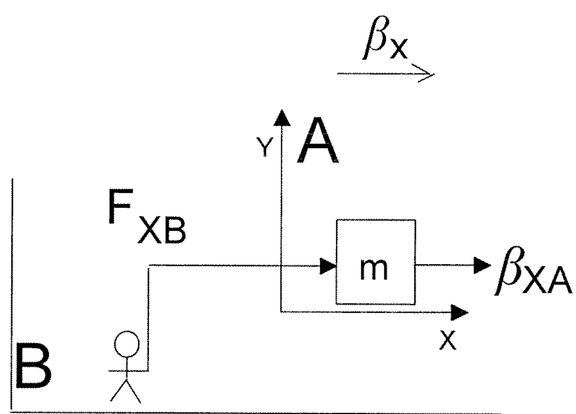

Figure 14

No momentum change of frame B occurs during this experiment, as a mirror image experiment with exactly opposite forces is run simultaneously by frame B, but is not shown. Both frames see mass m accelerate and both apply (20) and get the results:

$$F_{XA} = \left(\frac{mcA_{XA}}{(1-\beta_{XA}^2)^{3/2}} \right) \tag{44a}$$

$$F_{XB} = \left(\frac{mcA_{XB}}{(1-\beta_{XB}^2)^{3/2}} \right) \tag{44b}$$

F_{XA} is the force that frame A would have to apply to mass m to get exactly the same acceleration as is occurring with the application of force F_{XB}. Equation (14) can be applied to this experiment to give:

$$\beta_{XB} = \left(\frac{\beta_{XA} + \beta_X}{1 + \beta_X \beta_{XA}} \right) \tag{45}$$

Equation (18) can be applied to this experiment to give:

$$A_{XB} = A_{XA} \left(\frac{(1 - \beta_X^2)^{3/2}}{(1 + \beta_X \beta_{XB})^3} \right) \tag{46}$$

Inserting (45) and (46) into (44b) gives:

$$F_{XB} = \left(\frac{mcA_{XA}}{(1 - \beta_{XA}^2)^{3/2}} \right) = F_{XA} \tag{47}$$

Equation (47) has a meaning that is slightly different than the meaning of (23a). Equation (23a) says that the force that mass m "feels" (in a reference frame in which it is instantaneously stationary) is the same as the force applied to it by frame B. Equation (47) says that any reference frame traveling in the direction of the applied force F_{XB} will interpret the motion of mass m as being accelerated by a force equal to F_{XB}. In other words, F_{XB} is a constant to all reference frames traveling in the applied direction, regardless of their velocity.

The motion of mass m during the acceleration process now needs to be determined. First, from (47):

$$A_{XM} = \left(\frac{F_{XB}}{mc} \right) = \left(\frac{F_{XA}}{mc} \right) \tag{48}$$

Remembering that the relativistic acceleration is defined to be $A = a/c$ (where a is the Newtonian acceleration), A_{XM} is defined as the acceleration that mass m "feels". If mass m is traveling with velocity β_{XB} relative to frame B, the acceleration relative to frame B would be A_{XB}. From (18):

$$A_{XB} = A_{XM}(1 - \beta_{XB}^2)^{3/2} \tag{49}$$

If A_{XM} is a constant throughout the experiment, (49) would be true at every value of β_{XB} and at every frame B time t_B. Recognizing that $A_{XB} = d\beta_{XB}/dt_B$ gives the result:

$$A_{XM}(t_B + t'_B) = \left(\frac{\beta_{XB}}{\sqrt{1 - \beta_{XB}^2}} \right)$$

$$A_{XM} t'_B = \left(\frac{\beta_{XBO}}{\sqrt{1 - \beta_{XBO}^2}} \right) \tag{50}$$

β_{XBO} is the initial velocity of mass m at the starting time $t_B = 0$ of the experiment. Time t'_B is the constant resulting when (49) is integrated and is the time it would take mass m to accelerate from rest up to velocity β_{XBO}. This result leads to:

$$\beta_{XB} = \left(\frac{A_{XM}(t_B + t'_B)}{\sqrt{1 + A_{XM}^2(t_B + t'_B)^2}}\right) = \left(\frac{A_{XM}t_B + \left(\frac{\beta_{XBO}}{\sqrt{1-\beta_{XBO}^2}}\right)}{\sqrt{1 + (A_{XM}t_B + \left(\frac{\beta_{XBO}}{\sqrt{1-\beta_{XBO}^2}}\right))^2}}\right) \qquad (51)$$

Equation (51) gives the velocity of the mass relative to frame B as a function of frame B time, with the option of having the experiment start while mass m already has velocity β_{XBO}. Using $c\beta_{XB} = dx_B/dt_B$, equation (51) gives:

$$x_B = (c/A_{XM})\left[\left(1 + \left(A_{XM}t_B + \left(\frac{\beta_{XBO}}{\sqrt{1-\beta_{XBO}^2}}\right)\right)^2\right)^{1/2} - \left(\frac{1}{\sqrt{1-\beta_{XBO}^2}}\right)\right] + x_{BO} \qquad (52)$$

Frame B sees the position of the mass as x_B for any frame B time t_B with the option of having mass m start the experiment at coordinate x_{B0} instead of zero. Equations (48) through (52) give the motion of a mass with a constant acceleration relative to a stationary frame B.

The next issue to consider is illustrated in the experiment of Figure 15, where mass m starts

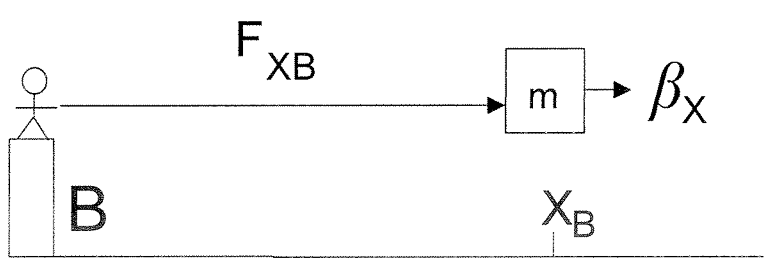

Figure 15

the experiment stationary in frame B. The observer applies force F_{XB} to mass m for a time period t_B and accelerates mass m to coordinate x_B. At that point, he stops pushing on m. A duplicate experiment is done simultaneously by frame B in the opposite direction so that frame B does not have a change in momentum during this experiment. This duplicate experiment is not shown.

Mass m starts the experiment at frame B coordinate zero and finishes accelerating at coordinate x_B. After this point, mass m travels at constant velocity β_X. During this experiment, the force F_{XB} is constant. This force that mass m feels must instantly stop when m reaches coordinate x_B. The energy output from the observer is $F_{XB}x_B$. This is the instant shown in Figure 15.

However, if the observer stops applying force F_{XB} at the instant mass m reaches coordinate x_B, mass m will continue accelerating until the "signal" of the force ending travels distance x_B and reaches mass m. This would take a time period x_B/c. This means mass m will keep accelerating past point x_B until the signal of the force ending reaches it. Mass m will then have a kinetic energy that is greater than $F_{XB}x_B$. The time period x_B/c that is required for the signal to travel to mass m is not trivial at the relativistic velocities of this experiment and the energy paradox stated above is significant.

The problem with this paradox is that the mechanism of force application is not clearly defined. For example, an easy solution to this paradox is to define the acceleration mechanism to be that shown in Figure 16. In this figure, mass m is shown at an intermediate position along the axis during the acceleration. There are a line of observers between the origin of frame B and position x_B. Each observer is only an incremental distance along the axis from the next in line. During the acceleration, each take a turn applying force F_{XB} to mass m. They pass mass m along the axis until it reaches position x_B. When the application of the force stops, there is no significant distance for the signal to travel and no resulting signal delay.

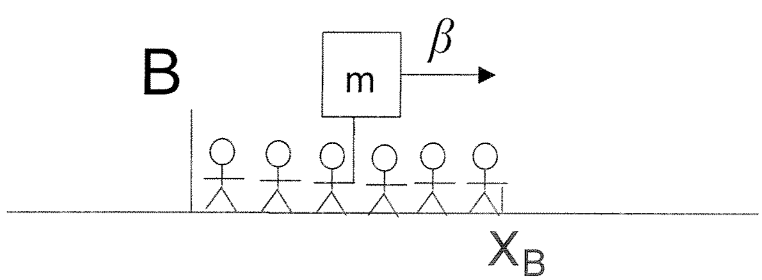

Figure 16

But another acceleration mechanism for experiment is also of interest. See Figure 17, which shows what is happening at an intermediate point during the acceleration. It still has a line of observers applying force F_{XB} sequentially, but this time they are pushing on the end of a pole. The length of the pole when stationary in frame B at the start of the experiment is x_B and, for this experiment, the pole will be assumed to be perfectly stiff and have no mass.

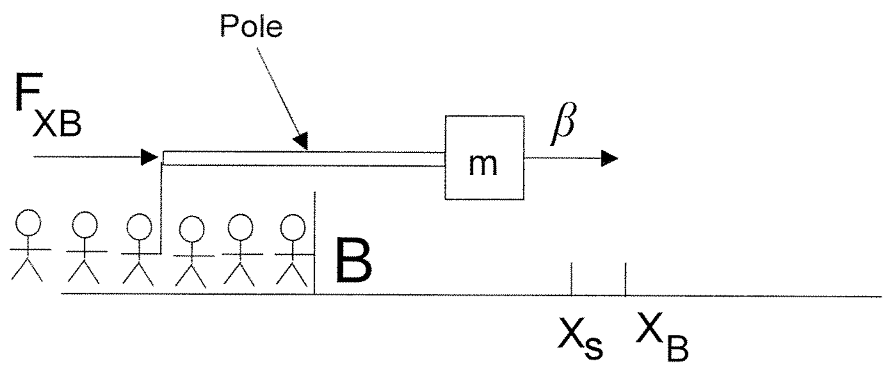

Figure 17

Near the end of this experiment, the observers will have pushed the pole up to the origin of frame B for a total distance of x_B. They will have expended energy $F_{XB}x_B$. When the end of the pole reaches the origin of frame B, the observers will stop applying the force and the signal of "no force" will start advancing up the pole until it reaches mass m. Because the pole is of length x_B, the time necessary for the signal to reach mass m is x_B/c.

The frame B time when the mass starts accelerating is defined to be zero. At the point where the end of the pole reaches the origin of frame B, the time on the clock of frame B is t_S and the mass m will be at coordinate x_S. The total time that it takes mass m to reach point x_B is t_B. Therefore:

$$t_B - t_S = x_B/c \tag{53}$$

As seen by frame B, at time t_S, the pole will be shorter than its original length x_B due length contraction. The instantaneous length of the pole results from a velocity distribution along its length but this velocity profile will not be covered in this article.

Of interest here is the fact that frame B will see mass m accelerate from position x_S to position x_B without an applied force. Although there is an explanation of these events presented in the preceding paragraphs, the frame B observers still make the observation that mass m can accelerate without any visible Newtonian force application (or reaction acceleration of another object). The frame A observers could easily conclude that the acceleration of mass m during the time interval between t_S and t_B is caused by a force that is acting on mass m. This force would be equal to F_{XB}.

This force fits the definition of a Mysterious Force as described in the article *Force and Geometry*. In that article, an applied force F_b perpendicular to the direction of motion of mass m produced a Mysterious Force F_{XA} (in the negative x-direction). In the experiment of Figure 17, an applied force in the direction of motion of mass m produces a Mysterious Force in the time-direction. There is a distortion of space-time geometry, and it shows itself as a force (or acceleration) existing at a time different from the time when the force was actually applied.

Frame B also sees a paradox at the beginning of the experiment. There is a time period x_B/c at the start of the experiment where a force application by frame B exists but mass m does not move. Although not precisely fitting the definition of Mysterious Force as defined previously, it is still a mysterious event. The net result of these two Mysterious Force events is that Newton's Law F = ma is not broken, but has been "delayed" by time interval x_B/c.

A third mechanism for this acceleration experiment is also of interest. See Figure 18.

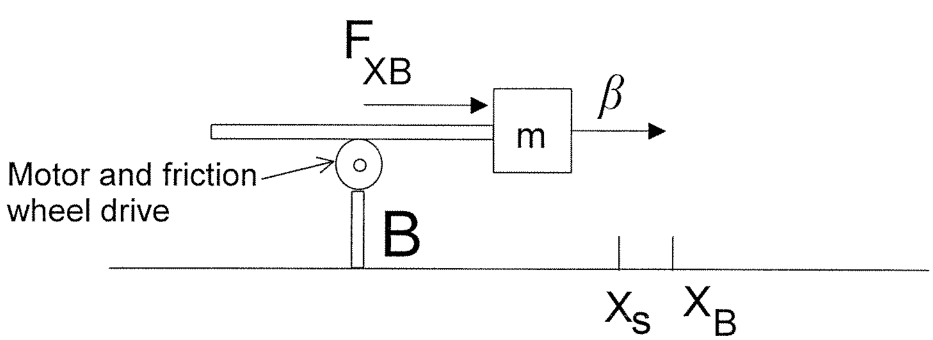

Figure 18

This experiment uses a motor and friction wheel located at coordinate zero of frame B to drive a pole pushing on mass m. At the start of the experiment, there is no time delay for the signal to reach mass m. But when mass m reaches coordinate x_S, the motor is shut off and there is a signal delay similar to the experiment of Figure 17. When mass m is at position x_S, the energy input by the motor is $F_{XB}x_B$. This is because the moving pole is now shorter (and is the same length as the pole in the experiment of Figure 17). This is when the motor stops and mass m continues accelerating for a time period x_B/c without a force applied by the motor. Mass m then has a kinetic energy equal to $F_{XB}x_B$. This experiment is most like the ideal experiment of Figure 15, but there is no energy paradox since a clear explanation of the accelerating mechanism is given.

In this case, Newton's Second Law is delayed only at the end of the experiment. The structure of the mechanism and geometry of the experiment affect the distortion of space-time and the appearance of Mysterious Forces. The delay in action-reaction caused by the time required for signals to travel various distances within the accelerating mechanism makes analysis of relativistic experiments more complex than analysis of Newtonian experiments. This delay produces Mysterious Forces, but these effects are not the main focus of this article. With the above stated understanding of signal delay effects, it can be seen that choosing an acceleration method similar to that shown in Figure 16 can result in an experiment where signal delay and its resulting Mysterious Force events are not present.

So, the following experiment will now be presented without showing the acceleration mechanism, but understanding that it is similar to that of Figure 16. This will eliminate any signal delay effects in the experiment.

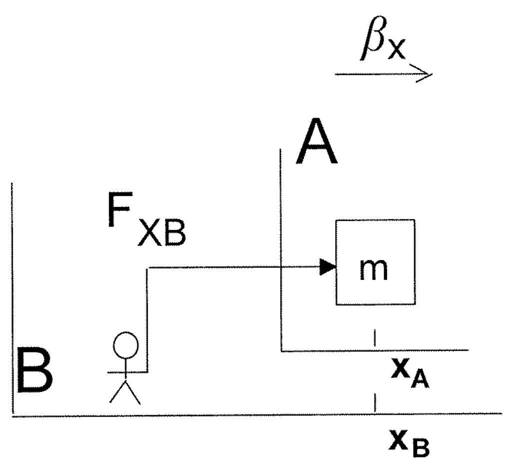

Figure 19

Figure 19 shows a mass m that is stationary in frame A at the end of the acceleration. Reference frame A has a velocity β_X in the direction of force application. Reference frame B is the source of the accelerating force F_{XB}. This force application stops as mass m reaches x_B and mass m continues at a constant velocity after this event. Mass m may also have a velocity at the beginning of the experiment but starts the experiment at coordinate zero in both reference frames. The motion of mass m is given by (47) through (52). When mass m reaches coordinate x_B (and x_A), frame B sees its own clock read t_B and the frame A clock that is next to mass m reads t_{AP} as given by adapting (11) as:

$$t_{AP} = \left(\frac{t_B - \left(\frac{x_B \beta_X}{c} \right)}{\sqrt{1-\beta_X^2}} \right) \tag{54}$$

But note that frame A sees the experiment continue after time t_{AP}. The reason for this is shown in Figure 20, where the complete experiment is depicted as frame A sees it.

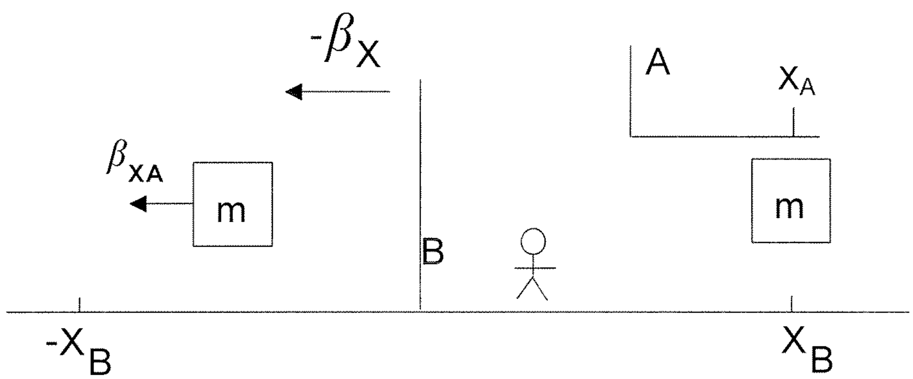

Figure 20

In Figure 20, the experiment is shown at the instant that mass m reaches coordinate x_B. Frame A sees frame B passing with velocity $-\beta_X$. Frame A also sees a second mass m with a velocity opposite in direction to that of the original mass m. The velocity and acceleration of this mass, as seen by frame B, is identical to that of the original mass except opposite in direction. Frame B also supplies the force to accelerate this mass. The acceleration of this second mass m is required to avoid a momentum change of frame B during the experiment.

Although both masses reach their respective coordinates of x_B and $-x_B$ simultaneously when viewed by frame B, frame A does not see this occur. Frame A sees the second mass m reach coordinate $-x_B$ at time t_{AP2} and:

$$\Delta t_{AP} = t_{AP2} - t_{AP} = \left(\frac{\left(\frac{2x_B \beta_X}{c} \right)}{\sqrt{1-\beta_X^2}} \right) \tag{55}$$

Frame A sees the experiment continue for time period Δt_{AP} after the first mass m reaches coordinate x_B. This is the time period when the second mass m is accelerating without an applied

Newtonian force (or reaction acceleration of some other mass). This time period appears to require a Mysterious Force to be acting on frame B to keep it from accelerating. One possible explanation that frame A might propose is that a gravitational field is acting on frame B for this time period. This is because no visible mechanism to apply force to frame B is seen. Or, alternatively, perhaps the gravitational field is acting on the second mass m. Time period Δt_{AP} does not result from "signal delay" because the method of accelerating the two masses has been chosen to eliminate this phenomenon.

The total duration of the experiment is t_B as seen by frame B, or $\Delta t_A = t_B\sqrt{1-\beta_X^2}$ as seen by frame A. Therefore, the portion of the total experimental time that the Mysterious Force is applied is given by:

$$\left(\frac{\Delta t_{AP}}{\Delta t_A}\right) = \left(\frac{\left(\frac{x_B\beta_X}{ct_B}\right)}{\sqrt{1-\beta_X^2}}\right)\left(\frac{2}{\sqrt{1-\beta_X^2}}\right) \tag{56}$$

Remembering that F_{XB} is equal to the force F_{XA} seen by frame A, equation (56) can also be written as:

$$\left(\frac{F_{XA}\Delta t_{AP}}{F_{XA}\Delta t_A}\right) = \left(\frac{-\left(\frac{x_B}{ct_B}\right)\beta_X}{\sqrt{1-\beta_X^2}}\right)\left(\frac{1}{\sqrt{1-\beta_X^2}}\right) \tag{57}$$

Equation (57) gives the total experimental momentum produced by the Mysterious Force divided by the total momentum of the experiment (which is defined to be the sum of the absolute value of the momentum change of both masses). Also, the minus sign has been added to show the direction of the momentum that would be produced by the Mysterious Force on frame B. In other words, frame A would expect frame B to accelerate in the positive direction as the second mass m is accelerated in the negative direction. Therefore, the Mysterious Force is in the negative direction to oppose this movement.

Equation (57) is manipulated in this way so that it can be compared to (32), which is rewritten as shown below.

$$\left(\frac{F_{XA}\Delta t_A}{F_{YA}\Delta t_A}\right) = \left(\frac{-\beta_{XB}\beta_{YB}}{\sqrt{1-\beta_X^2}}\right) \tag{58}$$

Equation (58) also gives the momentum produced by the Mysterious Force in the experiment of Figure 11 during a time interval Δt_A divided by the total momentum produced in that same time interval. In the experiment of Figure 11, frame A can see the tilted board acting on mass m and understands that the board applies a force to mass m in the negative x-direction. Frame A must also conclude that a Mysterious Force must act on frame B in the negative direction in order for frame B to apply the force F_{XA} to mass m without frame B itself accelerating.

The application of force to mass m (as observed by frame A) will distort the space-time geometry of the experiment. The Mysterious Force shown in the experiment of Figure 20 is produced by a combination of length contraction, time dilation and loss of simultaneity at a distance. These effects are the basis of (54) and are also the foundation of Special Relativity. These effects are also the basis of (32) and (33) in the article *Position, Velocity, Acceleration.*

To summarize this article, a force applied in the direction of motion of a mass does not change in magnitude when viewed by any other reference frame traveling in that same direction, regardless of its velocity. Also, two additional ways that Mysterious Forces are produced have been shown. These Mysterious Forces occur when a force is applied in the direction of motion of a mass, if the proper reference frame is chosen from which to view the experiment. These Mysterious Forces appear due to a distortion of the space-time geometry of the experiment.

To summarize this article and the article *Force and Geometry*, space-time distortion can be caused by a force perpendicular to or parallel to the direction of motion of a mass. This distortion results from the application of the Law of Conservation of Momentum (or Newton's Laws) to the experiment. This distortion is characterized by the effects of length contraction (the x-direction), time dilation and loss of simultaneity at a distance (the time-direction). These dynamic (inertial) experiments produce Mysterious Forces that are similar to gravitational force in many ways.