Although a lot of equations have been presented to show how Special Relativity affects the calculation of various quantities during acceleration, the actual concepts involved can be hidden by all the mathematics. These concepts become clearer when some specific examples are examined. Two cases involving the acceleration of objects with length (rods) will be presented. To simplify the numbers, a new length measurement, $z = 300,000,000$ meters, will be used. With $c = 1$ z/sec, the initial conditions for these cases are:

Case 1: length of rod $L = 0.5$ z, acceleration rate of 0.1 z/sec^2 ($a_B = 0.1$ sec^{-1}),
final velocity $\beta = 0.999$
Case 2: length of rod $L = 0.5$ z, acceleration rate of 10 z/sec^2 ($a_B = 10$ sec^{-1}),
final velocity $\beta = 0.5$

ROD LENGTH

As shown in Figure 9 from the article *The Acceleration Law*, objects A and B are positioned at the ends of an accelerating rod. Inertial frame i is watching the acceleration start from a velocity of zero at clock reading zero. While the acceleration is taking place, (13) helps define the coordinates of each object relative to their starting points (object B starts at frame i coordinate zero and object A starts at coordinate L). If the instantaneous velocity of the rod is defined to be the velocity of object B, then at any rod velocity β, $t_i = t_{oB}$, with t_{oB} defined by (14). Equations (14) and (21) define a_A. This means that $x_{iB} = x_{oB}$ per (15) but it also means that $x_{iA} \neq x_{oA}$ (because $t_i \neq t_{oA}$).

During the acceleration, as object B gets to velocity β, the length of the rod will be D^* and:

$$D^* = (x_{iA} + L) - x_{oB} \tag{70}$$

The actual values of D^* for the two cases are:

Case 1: $a_A = .09524$, $D^* = 0.022913$
Case 2: $a_A = 1.66667$, $D^* = 0.4873$

The length of the rod when traveling at a constant velocity β is D per (18a) and:

Case 1: $D = 0.022355$
Case 2: $D = 0.433013$

In either case, the inertial observers see that the length of the rod while accelerating is nearly equal to its length at a constant velocity. This is also true for a variety of other initial conditions. The velocity of the rod has the major impact on Special Relativity length contraction. Acceleration has a lesser effect.

If the acceleration of the rod stops when it reaches velocity β, frame i sees a gradual process of cessation of acceleration along the length of the rod. First, object B will stop accelerating at time t_{oB}. Due to its slower acceleration, object A has not reached this velocity yet. After object B gets

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to velocity β, this object will continue at this constant velocity as object A continues to accelerate until time t_{oA}. At t_{oA}, the entire length of the rod is traveling at constant velocity β.

Frame i sees the events of the acceleration happen earlier near the object B end of the rod and later near the object A end of the rod. The point where the rod achieves velocity β appears to sweep down its length going from the object B to object A. Once a location on the rod has reached velocity β, from that point forward it travels constantly at that velocity. This is summarized mathematically in (18b). For the two cases under consideration, (14) and (21) can be used to define D in (18b).

$$\text{Case 1: } t_{oA} = 234.611,\ t_{oB} = 223.439,\ D = 0.022355$$
$$\text{Case 2: } t_{oA} = 0.34641,\ t_{oB} = 0.057735,\ D = 0.433013$$

After the acceleration is over, the length of the rod is as (18a) predicts. But note that D^* is the length of the rod at the instant that it stops accelerating. Since $D^* > D$, the inertial observers see the rod shorten as the acceleration sweeps down the rod from object B to object A. Therefore, acceleration has the effect of reducing length contraction (as long as the acceleration is still occurring).

The view of the acceleration as seen by frame ii traveling by at velocity β is shown in Figure 10 of the article *The Acceleration Law*. This view is identical to the instantaneous view that observers on the rod have of the acceleration when the rod is stationary in frame ii. No length contraction at any velocity is seen by observers on the rod or any inertial frame traveling at the same instantaneous velocity as the rod.

CLOCK READINGS

Assume identical clocks are placed along the length of the rod and all these clocks read zero as the acceleration starts. The inertial frame will see the clocks read according to (16) while the acceleration is taking place. With the instantaneous velocity of the rod defined to be the velocity of object B, then at any velocity β, the clock reading of object B is $t'_{iB} = t'_{oB}$, with t'_{oB} defined by (17b). For the two cases:

$$\text{Case 1: } t'_{iA} = 39.390354,\ t'_{iB} = t'_{oB} = 38.002012$$
$$\text{Case 2: } t'_{iA} = 0.057646,\ t'_{iB} = t'_{oB} = 0.054931$$

In either case, the clock on object A is going faster than the clock on object B. This results from the fact that object B always has a velocity that is greater than object A during the acceleration. There is no direct experimental evidence of this time difference. However, in a gravitational acceleration field, experimental evidence shows that clocks at a higher potential (opposite in direction to the gravitational acceleration) do run faster than clocks at a lower potential.

Frame ii of Figure 10 sees the same clock readings. These are defined by (26). Frame ii sees the acceleration take longer for object A than it does for object B. Frame ii sees the objects stop accelerating simultaneously but does not see the objects start accelerating at the same time. The

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experiment actually starts for frame ii when the object A clock reads zero at frame ii time $t_{ii} = -\frac{L\beta/c}{\sqrt{1-\beta^2}}$ where object A is located at frame ii coordinate $x_{ii} = \frac{L}{\sqrt{1-\beta^2}}$. Object A has a velocity of $-\beta$ at the start of the experiment and will have a velocity of zero at frame ii time $t_{ii} = t_{oA} - \frac{L\beta/c}{\sqrt{1-\beta^2}}$. Object B has a velocity of $-\beta$ at frame ii time $t_{ii} = 0$ and will have a velocity of zero at frame ii time $t_{ii} = t_{oB}$. If these clock readings (when object velocities are zero as seen by frame ii) are plugged into (26), the result is (17). Frame ii sees the object B clock read t'_{oB} when the acceleration ends, just like frame i sees. Frame ii also sees the object A clock read t'_{oA} when the acceleration ends.

The difference between frame i and frame ii is that frame ii sees the objects start accelerating at separate times stop accelerating at the same instant. Frame i sees the reverse situation. The frame ii view of the experiment is the instantaneous view that the accelerating rod sees when it is at velocity β (relative to frame i). Observers on the accelerating rod see the acceleration of both objects start and stop simultaneously.

At the instant that the velocity of the rod is zero in frame ii, the clock readings on the objects are:

$$\text{Case 1: } t'_{oA} = 39.902112,\ t'_{oB} = 38.002012$$
$$\text{Case 2: } t'_{oA} = 0.329584,\ t'_{oB} = 0.054931$$

And using (14) and (21):

$$\text{Case 1: } t'_{oA} = t'_{oB}(1 + \tfrac{a_B L}{c}) = 39.902112$$
$$\text{Case 2: } t'_{oA} = t'_{oB}(1 + \tfrac{a_B L}{c}) = .329584$$

Therefore:

$$\frac{t_{oA}}{t_{oB}} = \frac{t'_{oA}}{t'_{oB}} = \frac{a_B}{a_A} = 1 + \frac{a_B L}{c} \tag{71}$$

It is now possible to define a "failure of simultaneity at a distance" $\Delta t'$ for an accelerating rod.

$$\Delta t' = t'_{oA} - t'_{oB}$$

$$\Delta t' = t'_{oB}\left(\tfrac{a_B L}{c}\right) = \tfrac{L}{c}\ln\left(a_B t_{oB} + \sqrt{1 + (a_B t_{oB})^2}\right)$$

$$\Delta t' = \tfrac{L}{c}\ln\left(\sqrt{\frac{1+\beta}{1-\beta}}\right) \tag{72}$$

Note that (72) does not depend on acceleration rate. Any acceleration path to velocity β gives the same $\Delta t'$ and gives the same length contraction.

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Frame ii and the accelerating rod see $\Delta t'$ directly. Frame i also sees the difference in clock readings on the accelerating rod as $\Delta t'$, but the reason for this is more complicated. First, frame i sees the difference on object clocks at the points where they reach velocity β. This difference is $\Delta t' = t'_{oA} - t'_{oB}$. But frame i also must see the normal "failure of simultaneity at a distance" given by (3). This means that frame i would think that the object A clock would be ahead of the object B clock by an amount $\Delta t' + L\beta/c$. However, since the objects do not get to velocity β simultaneously, this time difference will be reduced by the amount of time clock B advances before object A stops accelerating. This time interval is $\sqrt{1-\beta^2}\,(t_{oA} - t_{oB})$. From (27), this amount of time that clock B advances is (strangely enough) equal to the value for "failure of simultaneity at a distance" as seen by frame i. Therefore, $\Delta t' + L\beta/c - L\beta/c = \Delta t'$ as seen by frame i.

All of the observers in the experiment agree on the advance of the object A clock over the object B clock, regardless of their frame of reference. For the cases under review, these values are:

$$\text{Case 1: } \sqrt{1-\beta^2}\,(t_{oA} - t_{oB}) = 0.4995,\ \Delta t' = 1.9001$$
$$\text{Case 2: } \sqrt{1-\beta^2}\,(t_{oA} - t_{oB}) = 0.25,\ \Delta t' = 0.274653$$

In summary, the calculation of various quantities during an acceleration of an object with length is similar to the calculation of those quantities in the constant velocity experiment. Visually, the rod length and clock readings are modified somewhat from the constant velocity case. Length contraction is reduced during acceleration. Acceleration "failure of simultaneity at a distance" is similar to the constant velocity experiment, but in the opposite direction. Clocks at the higher potential position run faster than clocks at the lower potential position, similar to what occurs in a gravitational acceleration field.

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