A simple and accurate formula for the force of gravity is Newton's Gravitation Law.

$$F = \frac{Gm_1 m_2}{R^2} \tag{1}$$

F - Gravitational force between two masses
G - Gravitational Constant
m_1, m_2 - the two masses experiencing the force F
R - the distance between the two masses

THE UNIFORM LINEAR GRAVITATIONAL FIELD

The gravitational field of (1) is approximately spherically shaped around each of the masses. However, it will be useful to examine experiments using a gravitational field which is described by a straight line orthogonal coordinate system. This type of field is created using (1) and specifying that one of the masses is a pole of infinite length. The experiment is shown in Figure 1.

The gravitational field associated with this infinite pole is found by first specifying that mass m_2 is an infinitesimal mass dm within the pole. The individual gravitational fields of all the dm masses will then be added up to give the total gravitational field of the pole.

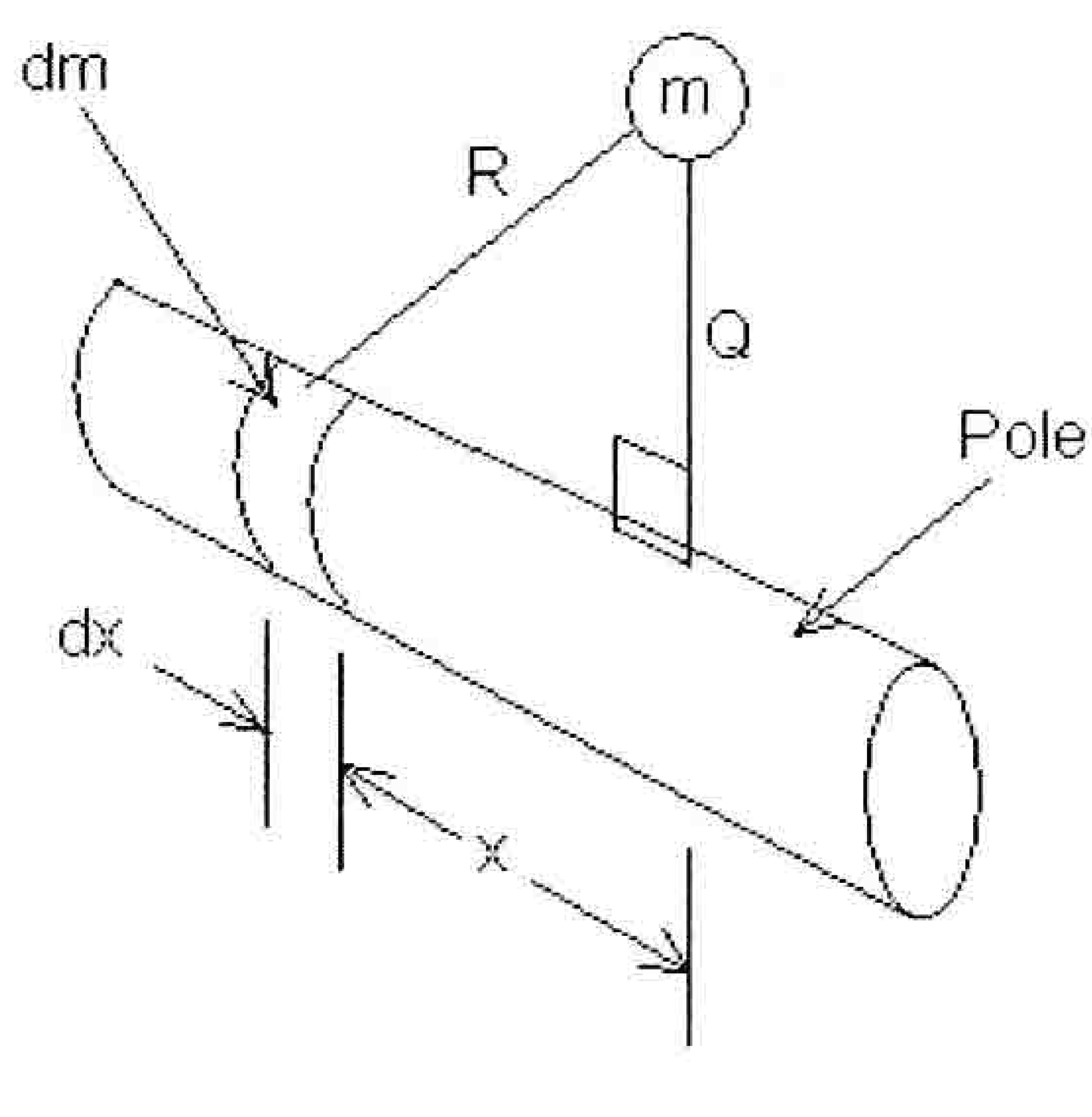

Figure 1. Infinitely long pole mass.

The pole has a density of δ and will have a resulting infinitesimal mass dm = δ dx . Infinitesimal mass dm is located a distance x from mass m. Mass m is located a distance Q above the pole. Using (1), the resulting gravitational force dF is:

$$dF = \frac{Gm\,\delta dx}{Q^2 + x^2} \tag{2}$$

When viewed in the Q-x plane, the force dF can be seen to contribute to the force df (in the Q direction) between mass m and the pole. This is shown in Figure 2.

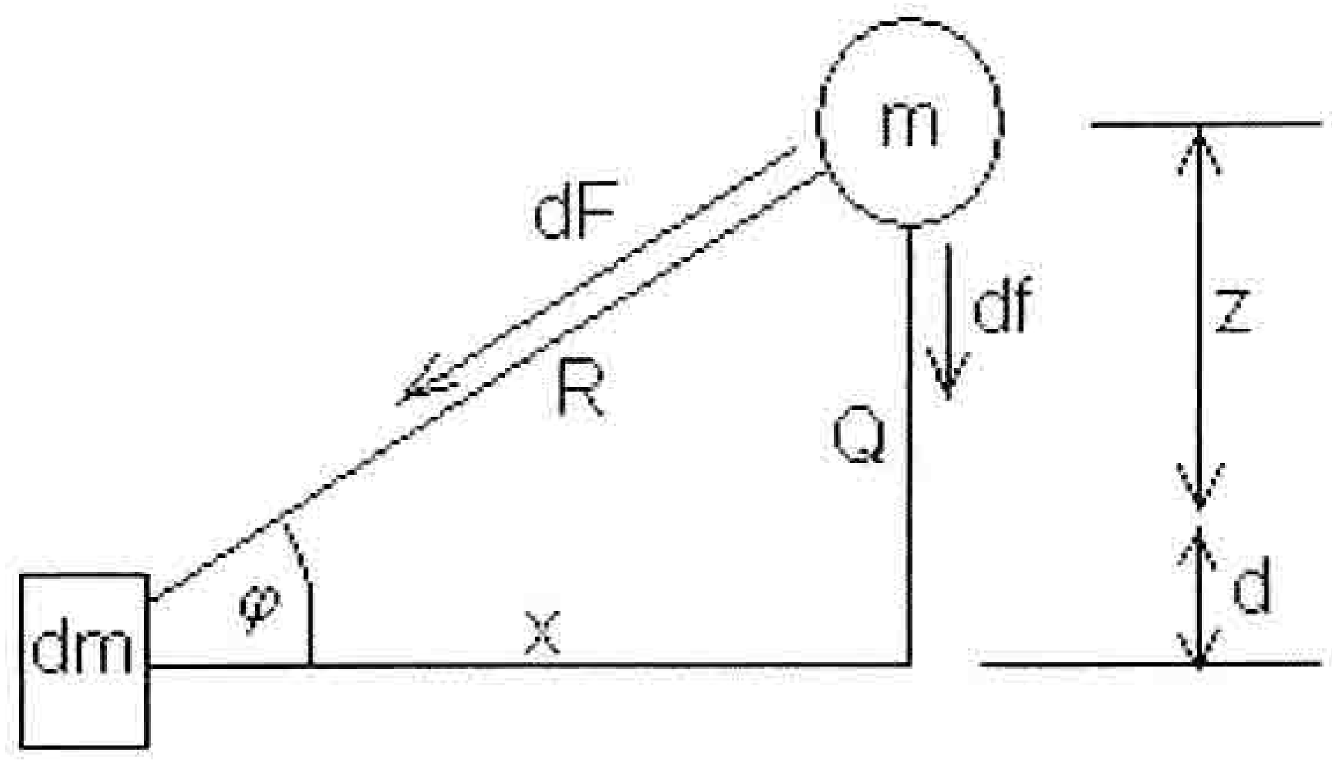

Figure 2. Q-x plane view of the forces.

The resulting equation for the force between mass m and the pole is:

$$df = dF \sin\phi = dF\left(\frac{Q}{\sqrt{Q^2 + x^2}}\right)$$

$$f = 2Gm\,\delta Q \int_0^\infty \frac{dx}{\left(Q^2 + x^2\right)^{3/2}}$$

$$f = \frac{2Gm\,\delta}{Q} = \frac{2Gm\,\delta}{z + d} \tag{3}$$

If a constant g is defined to be the gravitational acceleration divided by c, a new quantity of acceleration in the z direction can be defined as:

$$\frac{f}{2Gm\,\delta} = \frac{\frac{1}{d}}{\frac{z}{d} + 1} = \frac{g_z}{c} \qquad \text{for any z}$$

$$\frac{g_0}{c} = \frac{1}{d} \qquad \text{for the case z = 0}$$

$$g_z = \frac{g_0}{1 + \frac{g_0 z}{c}} \tag{4}$$

Note that $\delta = 6.733 \times 10^{26}$ kg/m will make $2G\delta / c = 1$. Equation (4) is a gravitational acceleration that is in the same form as the dynamic acceleration (21) in the article *The Acceleration Law*. The distance Q is divided into two parts, z and d, defined in Figure 2. At distance d, the acceleration g_0 is equivalent to α From (21). At distance z above d, the acceleration g_z is equivalent to acceleration α_L from (21) with L = z .

This special gravitational field will be referred to as a Dynamic Equivalent Orthogonal Gravitational Field (DEOGF). It is orthogonal because the force of gravity exists uniformly along the x-axis in the z direction. All experiments in the x-z plane simulate a dynamic acceleration in the z direction.

FORCE TRANSFORMATION IN A GRAVITATIONAL FIELD

Consider a thought experiment where an object B has mass m and is stationary in reference frame B at a distance d above the pole mass. An identical object A is stationary in reference frame A at a distance z above frame B. Since the gravitational acceleration of objects A and B is different, then the gravitational force felt by someone holding the objects would be different. These forces are:

$F_B = mg_B c$ for object B $\qquad$ $F_A = mg_A c$ for object A

$$F_A = F_B \left(\frac{g_A}{g_B} \right) = \frac{F_B}{1 + \frac{g_B z}{c}} \tag{5}$$

Now assume that object A falls from frame A to frame B. The kinetic energy generated by this fall is KE. To find KE, note that object A becomes an inertial reference frame as soon as it starts to fall. Object B accelerates towards object A with constant acceleration g_B . If the time interval that object A sees for the fall is t_A' , then:

$$z = \frac{c}{g_B} \left[\left(1 + (g_B t_A')^2 \right)^{1/2} - 1 \right]$$

$$g_B t_A' = \left[\left(1 + \frac{z g_B}{c} \right)^2 - 1 \right]^{1/2} \tag{6}$$

The velocity that object A has relative to object B at the moment of impact is β_A and :

$$\beta_A = \frac{g_B t_A'}{\sqrt{1 + (g_B t_A')^2}} \tag{7}$$

The kinetic energy of object A as it impacts frame B is:

$$KE = \frac{mc^2}{\sqrt{1 - \beta_A^2}} - mc^2$$

$$\frac{1}{\sqrt{1 - \beta_A^2}} = 1 + \frac{z g_B}{c}$$

$$KE = mczg_B \tag{8}$$

Now lets assume object A does not fall from frame A. Instead, it is lowered by observer B with a pole to frame B. Observer B does not know what force he will feel on his end of the pole. He therefore assumes this force will vary with z and calls it F(z). The work gained by observer B during this task will be W_{AB}.

$$W_{AB} = \int_z^0 F(z)dz \tag{9a}$$

$$W_{AB} = F_{ave} z \tag{9b}$$

The value of F(z) would be applied for an incremental distance dz to give the expression for work in (9a). Another way to calculate the work would be to take the average force during the task F_{ave} and multiply it by z, as is shown in (9b). But, knowing $KE = W_{AB}$ gives:

$$F_{ave} = mcg_B = F_B \tag{10}$$

Equation (10) is true no matter what the value of z is. For any value of z, observer B always feels average force F_B on his end of the pole. Therefore, observer B always feels constant force F_B on his end of the pole. This force will be called F_{BA}, and object A exerts force F_A on the opposite end of the pole.

$$F_{BA} = F_A\left(\frac{g_B}{g_A}\right) = F_A\left(1 + \frac{g_B z}{c}\right) = F_B \tag{11}$$

THE FORCE OF GRAVITY ON ENERGY

Consider an experiment where a mass is moved from frame A to frame B using a pole. The weight of the mass in frame B is F_B. This movement happens slowly so that no dynamic effects are present. The work received from this experiment is W_{AB} and:

$$W_{AB} = F_B z \tag{12}$$

In frame B, after the mass has arrived, some of the material of the mass is converted into heat energy H (using nuclear fission) and held inside of the mass with insulation. Then, as a separate activity, energy H could be extracted from the mass and combined with W_{AB}. The total energy gained from the experiment is $W_{AB} + H$.

Now the experiment is repeated, but this time the identical material conversion to heat energy H is made in frame A. This energy is again held inside of the mass. The mass is again moved from frame A to frame B. At the frame B location, the energy H is extracted from the mass and added to the energy obtained from the movement of the mass from frame A to frame B. The total energy must again be $W_{AB} + H$ from the Law of Conservation of Energy. The beginning and end states of the experiment are the same, so the total energy received from the experiment must be the same. If W_{AB} is the same for both experiments, then the force of gravity on the energy H must be the same as the force of gravity on the material that was converted into H. Similar experiments could be performed on any other type of energy with the same result.

HORIZONTAL FORCE IN A GRAVITATIONAL FIELD

In Figure 3, observer B pushes on a long board with a hole in it so that it passes around the pole mass. The same board is part of an identical experiment on the opposite side of the pole mass (not shown), which is positioned to eliminate any rotational movement or torque effects in the experiment. Horizontal force F_B is applied by observer B and horizontal force F_A is felt by observer A in frame A. The spring in frame A is compressed, clamped in the compressed position and lowered to frame B by observer B using a pole.

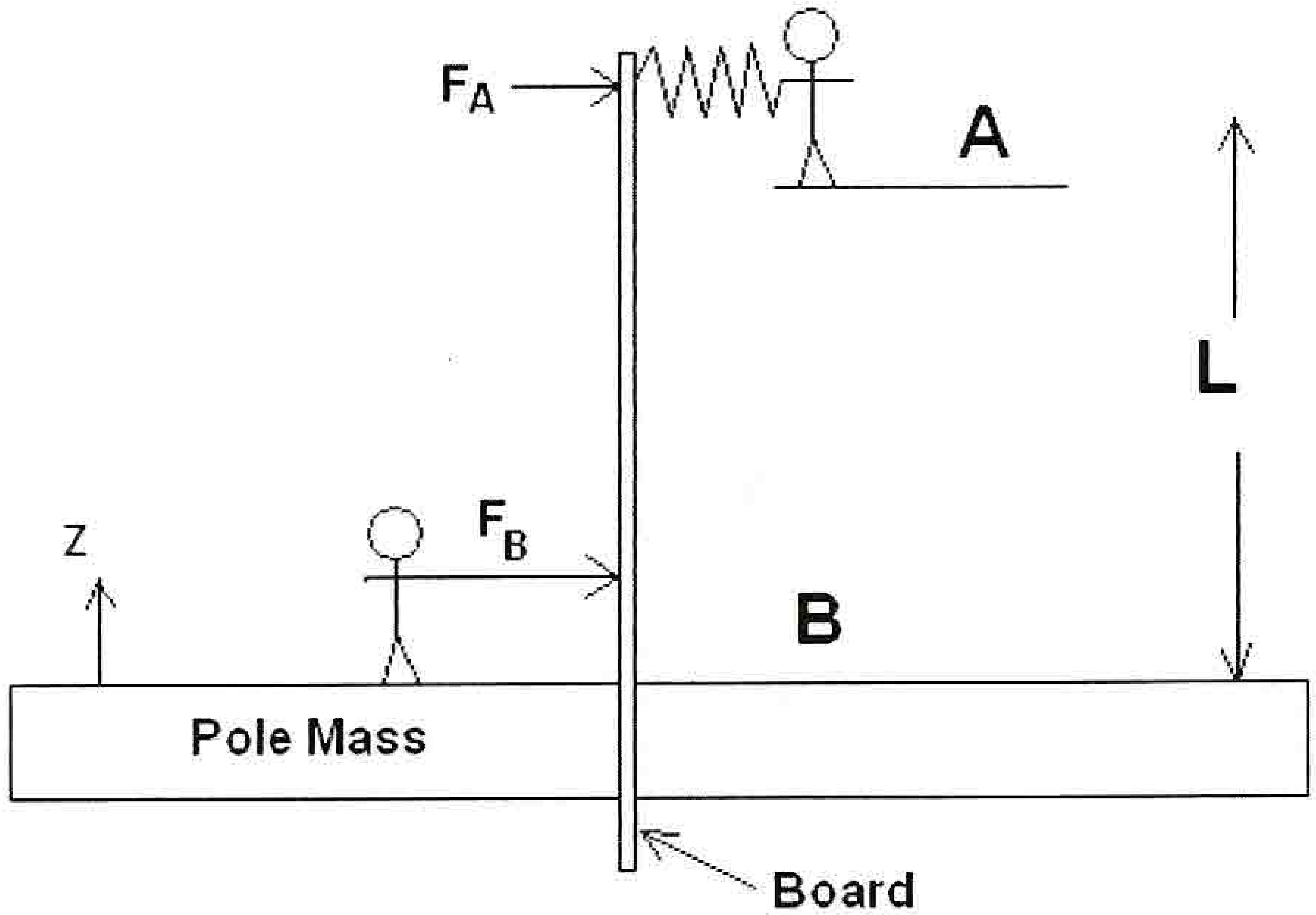

Figure 3. Horizontal force experiment.

The spring has a force-displacement relationship of $F = kdx$, where k is the spring constant and dx is the displacement. When the board compresses the spring, a potential energy E is stored in the spring while in frame A. When the spring has been moved to frame B, the same energy and spring force must be present there. The work gained by lowering the spring down to frame B is:

$$W_{AB} = \left(\frac{E}{c^2}\right) g_B cL \tag{13}$$

The energy initially expended by observer B on the board is $\frac{1}{2} F_B dx$ and the energy stored in the spring is $E = \frac{1}{2} F_A dx$. When the spring is at frame B, the Law of Conservation of Energy gives:

$$\frac{1}{2} F_B dx = E + W_{AB} = \frac{1}{2} F_A dx + W_{AB}$$

$$\frac{F_B}{F_A} = 1 + \frac{g_B L}{c} = 1 + \frac{W_{AB}}{E} \tag{14a}$$

$$g_A = \frac{g_B}{1 + \frac{W_{AB}}{E}} \tag{14b}$$

In (14a), the relationship for horizontal force is seen to be the same as the relation for force in the direction of the gravity field (5). Equation (14b) shows that the Acceleration Law (4) can also be thought of as a function of the energies in the experiment.

TIME IN A GRAVITATIONAL FIELD

The transformation for time flow rates at different gravitational potentials is found from the experiment shown in Figure 4, where a board once again moves horizontally over the pole mass. However, this time the frame B observer applies a force F_B to the board and simultaneously to a second mass in frame B going in the opposite direction. The mass in frame A will acquire a velocity to the right and the mass in frame B will acquired a velocity to the left. The experiment starts with all clocks reading zero and the total system momentum being zero.

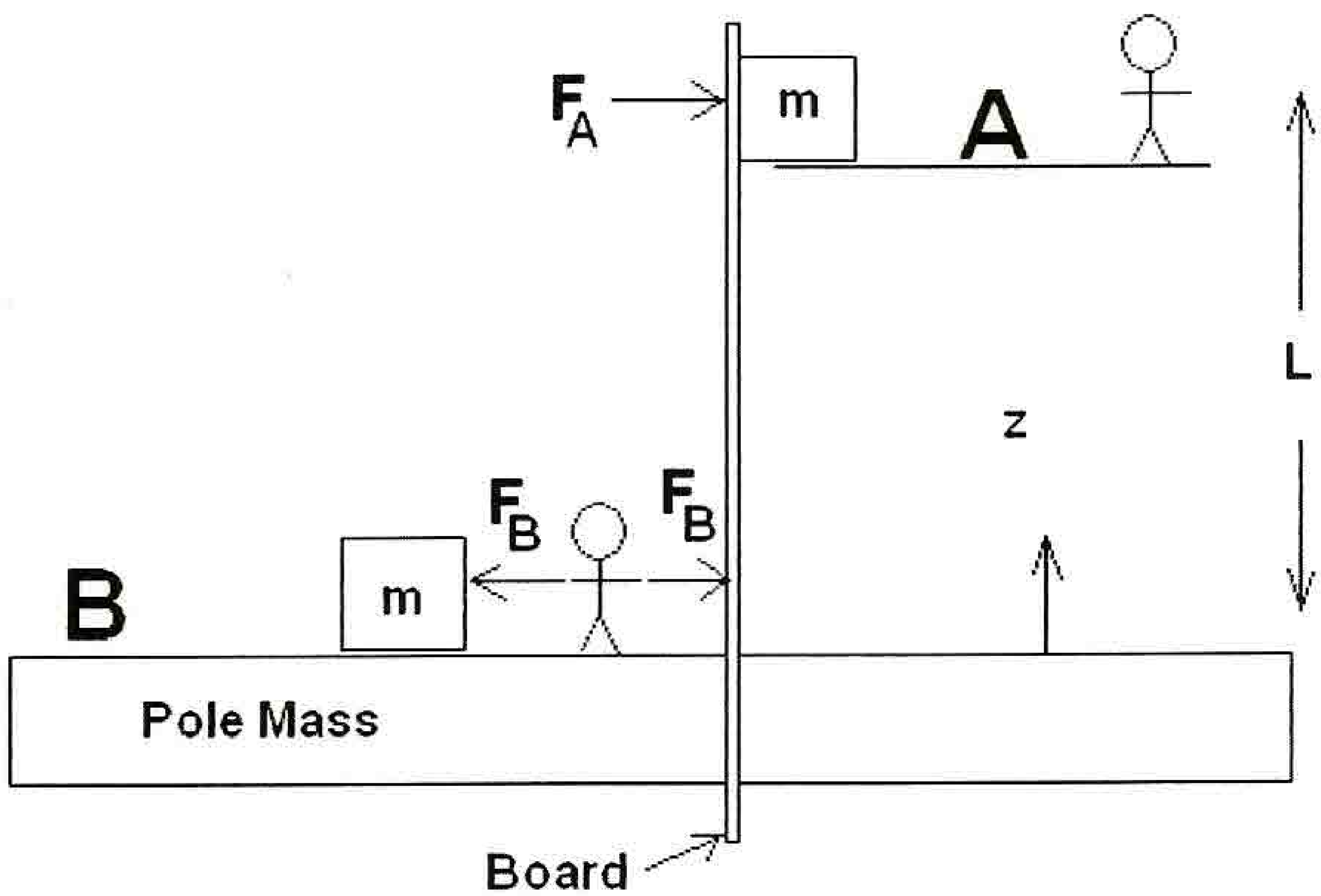

Figure 4. Time experiment.

Observer B applies the forces until clock reading t_B and the frame A observer sees the force at his location applied until clock reading t_A. The momentum produced in frame B must be equal in magnitude (and opposite in direction) to the momentum produced in frame A.

$$F_A t_A = F_B t_B$$

$$\frac{t_A}{t_B} = 1 + \frac{g_B L}{c} \qquad (15)$$

Equation (15) can be compared to the equation (31) or (34a) from the article *The Acceleration Law* :

$$\frac{t_{iiA}'}{t_{iiB}'} = 1 + \frac{\alpha_B L}{c} \qquad \text{(31: \textit{The Acceleration Law})}$$

The clock readings at different levels in a gravitational field vary in the same way as the clock readings in an equivalent dynamic experiment. Note that "length contraction" is not assumed to occur in the direction of gravity or in the direction perpendicular to gravity. There are two reasons for doing this. First, the best assumption in any experiment is the simplest one and no length contraction is the simplest possible assumption for length behavior in these experiments.

Second, in the article *The Acceleration Law*, it was shown that an object of length L undergoing acceleration (as described by (21) of that article) does not see length contraction in the accelerating frame of reference. Length contraction has therefore been established to be absent when acceleration is involved and is only present when there is a relative velocity between reference frames. This dynamic precedent is assumed to apply to the case of gravitational acceleration too. Since (4) and (21: *The Acceleration Law*) are similar, the article *Acceleration Dynamics* applies equally to a gravitational acceleration and a dynamic acceleration.

SUMMARY

This article has presented thought experiments that result in transformations for length, force and time in a DEOGF. These transformations can be expressed as functions of the gravitational potential energy difference between reference frames or as functions of a local gravitational acceleration. The DEOGF is exactly equivalent to a dynamic acceleration. This is an exact demonstration of Einstein's Equivalence Principle, a thought experiment influencing the development of the General Theory of Relativity.