A simple and reasonably accurate formula for the force of gravity is Newton's Gravitation Law.

$$F = \left(\frac{Gm_1 m_2}{R^2}\right) \tag{1}$$

F - Gravitational force between two masses

G - Gravitational Constant

m_1, m_2 - the two masses experiencing the force F

R - the distance between the two masses

Equation (1) describes approximately spherically shaped gravitational fields that surround point masses m_1 and m_2. However, the experiments in this article will use an orthogonal gravitational field, which is a gravitational field described by a right angle straight line coordinate system with the vertical z-direction as the direction of gravitational force F. This orthogonal gravitational field is created by using (1) and specifying that one of the masses of the experiment is a pole of infinite length. This pole is shown in perspective view in Figure 1.

The gravitational field associated with this infinite pole is found by first specifying that mass m_2 from (1) is an infinitesimal mass dm within the pole. The individual gravitational fields of all of these infinitesimal dm masses will be added up to give the total gravitational field of the pole.

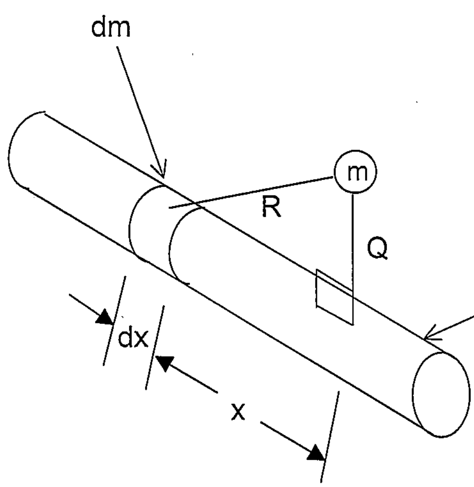

Figure 1

The pole has a density of δ and will have a resulting infinitesimal mass dm given by dm = δ dx as shown in Figure 1. Infinitesimal mass dm is located a distance x from mass m. Mass m is located a distance Q above the pole. Using (1), the resulting gravitational force dF for this arrangement is:

$$dF = \left(\frac{Gm\delta dx}{Q^2 + x^2}\right) \qquad (2)$$

When viewed in the x-Q plane, the force dF can be seen to contribute to the force df (in the Q direction) between mass m and the pole. This is shown in Figure 2.

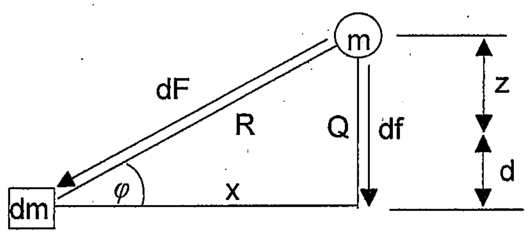

Figure 2

This results in:

$$df = dF\sin\varphi = dF\left(\frac{Q}{\sqrt{Q^2 + x^2}}\right) \qquad (3)$$

The resulting equation for the force between mass m and the pole is:

$$f = 2Gm\delta Q \int_0^\infty \left(\frac{dx}{(Q^2 + x^2)^{3/2}}\right)$$

$$f = \frac{2Gm\delta}{Q} = \frac{2Gm\delta}{z + d}$$

$$\frac{f}{2Gm\delta} = \frac{\frac{1}{d}}{\frac{z}{d} + 1} = \frac{g_z}{c} \text{ where } g \text{ is } \frac{\text{gravitational acceleration}}{c}$$

$$\frac{g_o}{c} = \frac{1}{d} \text{ for } z = 0$$

$$g_z = \frac{g_o}{1 + \frac{g_o z}{c}} \qquad (4)$$

Note: $\delta = 6.733 \times 10^{26}$ kg/m, so that $2G\delta/c^2 = 1$.

In (4), the quantity $\frac{f}{2Gm\delta}$ has the units of $\frac{1}{d}$. These units are also the units of $\frac{\text{acceleration}}{\text{velocity}^2}$. So g is a new quantity defined to be a gravitational acceleration that is normalized to the speed of light. This allows (4) to be put in a form analogous to (21) from the article *The Acceleration Law*. The distance Q is made of two parts, z and d, as defined in Figure 2. At distance d, the acceleration g_o is equivalent to a from (21). At distance z above d, the acceleration g_Z is equivalent to acceleration a_L from (21) (with L = z).

This special gravitational field with be referred to as a Dynamic Equivalent Orthogonal Gravitational Field (DEOGF). It is called 'orthogonal' because the force of gravity only exists in the z-direction and is uniform in the x direction. All experiments in the x-z plane simulate a dynamic acceleration in the z direction.

In the article *The Acceleration Law*, it was shown that an object of length L undergoing an acceleration described by (21) of that article does not see length contraction in the accelerating frame of reference. It maintains length L constantly even though the acceleration of each end of the object is different. Length contraction is seen by inertial reference frames with a relative velocity watching the acceleration. An object standing in a gravitational field of the form (4) above would therefore not expect to see any length contraction in the direction of gravitational acceleration.

Consider a thought experiment where an object B is stationary in reference frame B at distance d from the pole mass. An object A is stationary in reference frame A at distance z above frame B. Both objects have the same mass value m. Since the gravitational acceleration of objects A and B is different, then the gravitational force F that each object "feels" in its local reference frame is different. If g_A is the acceleration of object A and g_B is the acceleration of object B, then:

$F_B = mg_Bc$ for object B $\qquad$ $F_A = mg_Ac$ for object A

$$F_A = F_B\left(\frac{g_A}{g_B}\right) = \left(\frac{F_B}{1+\frac{g_Bz}{c}}\right) \qquad (5)$$

Equation (5) applies for forces in the direction of gravitational acceleration. Now assume that object A falls from frame A and impacts on frame B. The kinetic energy generated by this fall is KE. To find KE, note that object A becomes an inertial reference frame as soon as it starts falling. Object B accelerates towards object A with constant acceleration g_B. If the time interval that object A sees for the fall to take place is t'_A, then:

$$z = \frac{c}{g_B}[(1+(g_Bt'_A)^2)^{1/2} - 1]$$

$$g_Bt'_A = [(1+\frac{zg_B}{c})^2 - 1]^{1/2} \qquad (6)$$

The velocity that object A has relative to object B at the moment of impact is β_A and:

$$\beta_A = \frac{g_B t'_A}{[1 + (g_B t'_A)^2]^{1/2}} \tag{7}$$

The kinetic energy of object A as it impacts frame B is:

$$KE = \frac{mc^2}{\sqrt{1 - \beta_A^2}} - mc^2$$

$$\frac{1}{\sqrt{1 - \beta_A^2}} = 1 + \frac{zg_B}{c}$$

$$KE = mczg_B \tag{8}$$

Now lets assume object A does not fall from frame A. Instead it is lowered by observer B with his pole to frame B. The work gained by observer B during this task will be W_{AB} and:

$$W_{AB} = \int_z^o F(z)dz \tag{9a}$$

$$W_{AB} = F_{ave} z \tag{9b}$$

Observer B does not know what value of force he will feel on the end of his pole as he lowers object A through the range of values of z. He therefore assumes this force will vary with z and calls it $F(z)$. The value of $F(z)$ would be applied for incremental distance dz to give the expression for W_{AB} shown in (9a). Another way to calculate W_{AB} would be to take the average force F_{ave} and multiply it by z, as is shown in (9b). But knowing that $KE = W_{AB}$ gives:

$$F_{ave} = mcg_B = F_B \tag{10}$$

No matter what value of z is, F_{ave} is equal to a constant, F_B. No matter what the length z of the pole is, the force that observer B feels on his end of the pole is always F_B. The observer B feels this force, which will be called F_{BA}, and object A exerts force F_A on its end of the pole.

$$F_{BA} = F_A\left(\frac{g_B}{g_A}\right) = F_A(1 + \frac{g_B L}{c}) = F_B \tag{11}$$

Now, consider an experiment where a mass is moved from frame A to frame B using a pole. This happens slowly so that dynamic effects do not significantly affect the experiment. The work received from this change in position is W_{AB}:

$$W_{AB} = F_B z \tag{12}$$

The weight of the mass in frame B is F_B. In frame B, after the mass has arrived, some of the material of the mass could be converted into heat energy H (using nuclear fission) and held inside of the mass with insulation. Then, as a separate activity, energy H could be extracted from the mass and combined with W_{AB}. The total energy received from the experiment would be W_{AB} + H.

Now the experiment is repeated, but this time the identical material conversion to heat energy H is made in frame A. This energy is again kept inside of the mass. The mass is again moved from frame A to frame B. At the frame B location, the energy H is extracted from the mass and combined with the energy obtained by the movement from frame A to frame B. The energy received by the movement from frame A to frame B must also be W_{AB}, giving a total energy of W_{AB} + H once again. Any other result would violate the Law of Conservation of Energy. The two experiments have beginning and end states that are the same, so the total energy received from the two experiments must be the same. Therefore the weight of energy H caused by the acceleration must be the same as the weight of material mass that was converted into energy H.

For an experiment showing the relationship of forces applied perpendicular to the direction of gravitational acceleration, see Figure 3. Observer B pushes on a long board with a hole in it that allows it to pass around the infinitely long pole to reach the opposite side. The board is connected to an identical experiment (not shown) on the opposite side of the pole, positioned to eliminate any moment (torque) effects in the experiment. Force F_B is applied by observer B but force F_A is felt by observer A. The spring in frame A is compressed, clamped in the compressed position and relocated to frame B by observer B using a pole.

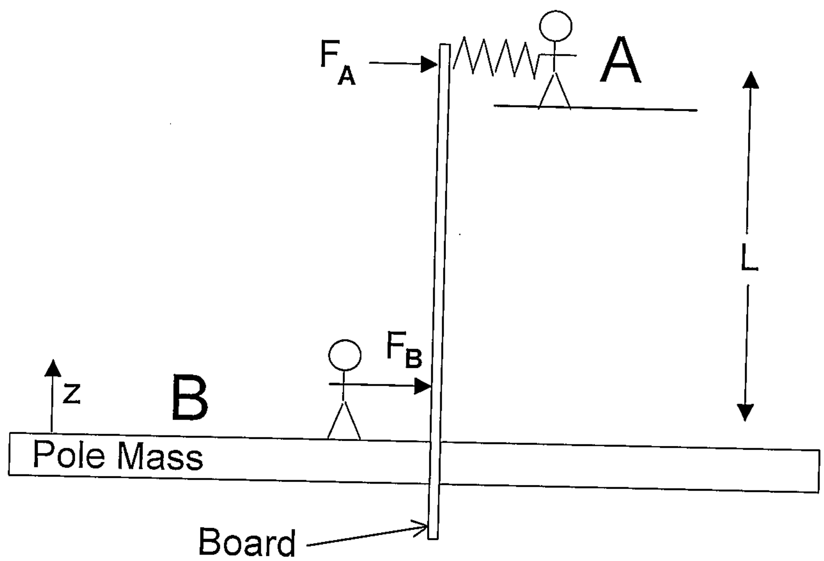

Figure 3

The spring has a force-displacement relationship of F = kdx (k - spring constant, dx - spring displacement). When the board compresses the spring, this results in a spring potential energy of E in frame A. When the spring has relocated to frame B, its potential energy in frame B is still E. The work gain resulting from lowering the spring energy down to reference frame B is W_{AB}.

$$W_{AB} = \left(\frac{E}{c^2}\right) g_B c L \tag{13}$$

The energy contained in the spring is $E = \frac{1}{2} F_A dx$. The energy expended initially by observer B is $\frac{1}{2} F_B dx$. With the spring energy moved down to frame B, the Law of Conservation of Energy gives:

$$\frac{1}{2} F_B dx = \frac{1}{2} F_A dx + W_{AB} = E + W_{AB}$$

$$\frac{F_B}{F_A} = 1 + \frac{W_{AB}}{E} = 1 + \frac{g_B L}{c} \tag{14a}$$

$$g_A = \frac{g_B}{1 + \frac{W_{AB}}{E}} \tag{14b}$$

Equation (14) shows that the Acceleration Law (4) can also be thought of as a function of the potential energy W_{AB}.

The transformation for time at different gravitational potentials is found from an experiment that is similar to the one of Figure 3. In Figure 4, the board once again passes around the infinitely long pole for the same reason as it did in the experiment of Figure 3. However, this time the frame B observer applies a force F_B to the board and simultaneously to a second mass m located in frame B. The mass in frame A will acquire a velocity to the right. The mass in frame B will travel in a direction to the left. The experiment starts with all clocks reading zero and the total momentum of the system being zero. Observer B applies the forces until clock reading t_B and the observer in frame A sees the force at his location applied until clock reading t_A. The momentum produced in frame B must be equal in magnitude (and opposite in direction) to the momentum produced in frame A.

$$F_A t_A = F_B t_B$$

$$\frac{t_A}{t_B} = 1 + \frac{g_B L}{c} \tag{15}$$

Equation (15) can be compared to the equation (31) or (34a) in *The Acceleration Law:*

$$\frac{t'_{iiA}}{t'_{iiB}} = 1 + \frac{a_B L}{c} \qquad \text{(31: \textit{The Acceleration Law})}$$

The clock readings at different levels in the gravitational field vary in the same way as the clock readings with equivalent dynamic accelerations.

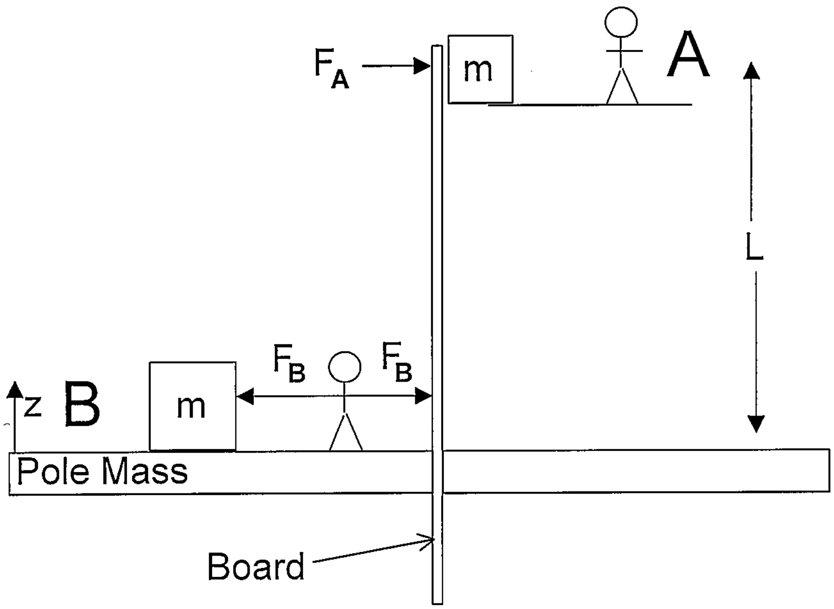

Figure 4

Since (4) and (21) from *The Acceleration Law* are similar, the article *Acceleration Dynamics* applies equally to a gravitational field and a dynamic acceleration.

In summary, this article has presented thought experiments that result in transformations for length, force and time in a Dynamic Equivalent Orthogonal Gravitational Field. These transformations can be expressed as a function of the gravitational potential energy difference between reference frames located within that field. The DEOGF is exactly equivalent in every way to a dynamic acceleration. This is an exact mathematical demonstration of Einstein's Equivalence Principle, a thought experiment influencing the development of the General Theory of Relativity.

One final speculation for this discussion has to do with fields in general. The analysis presented for gravitational fields might be applied to other types of fields. Magnetic, electrostatic or any other field (where potential energy is stored due to a movement of an object within the field) should give relativistic transformations for length, time, force. Since mass may not be the source of attraction in other fields, mass may have to be replaced by another quantity in the transformations (such as charge).