

The article *The Real Ladder Paradox* showed how the acceleration of an object with length can be influenced by the Law of Conservation of Energy. This should be just a geometry analysis, so why is energy involved? This is an indication that energy and geometry are linked at some fundamental level. To explore this idea further, the length contraction process during acceleration will be examined in detail.

Definition of Terms

Figure 8 shows an object accelerating relative to inertial reference frames i and o.

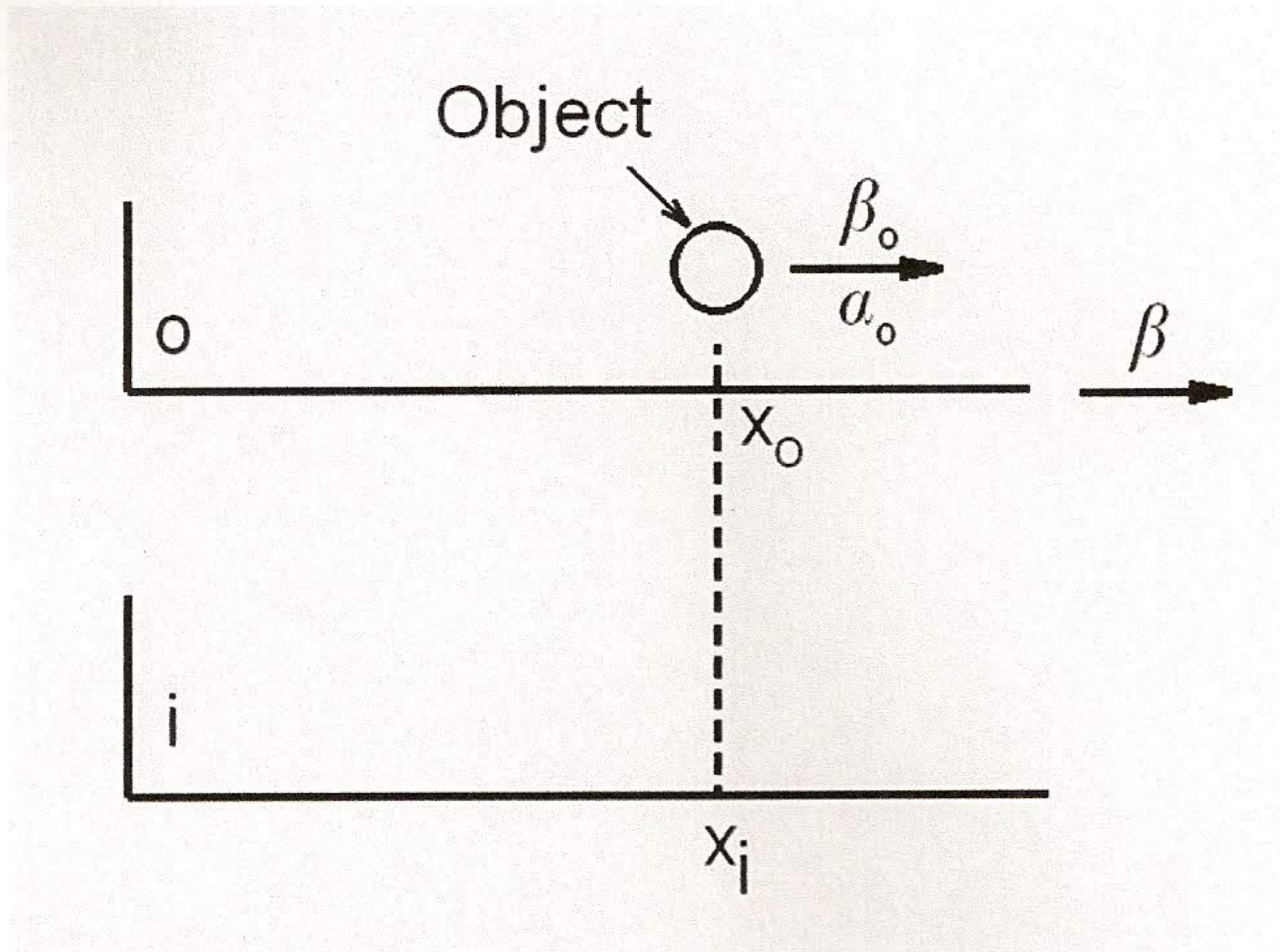


Figure 8. Single point object accelerates relative to frame i.

Frame o is moving with velocity β relative to frame i. Constant acceleration α_o is 'felt' by observers on the object. The acceleration starts from coordinate zero in frame i and at time zero for clocks on the object and both reference frames. The speed of light is c. The acceleration of the object relative to frame i can be found from the familiar velocity addition formula of Special Relativity. If β_i and β_o are the relativistic velocities of the object relative to frame i and o, then:

$$\beta = \frac{\text{velocity}}{c}$$

$$\alpha = \frac{\text{acceleration}}{c}$$

$$\beta_i = \frac{\beta + \beta_o}{1 + \beta\beta_o} \quad (9)$$

Differentiating (9) gives:

$$\frac{d\beta_i}{dt_i} = \frac{d\beta_o}{dt_o} \left[\frac{1 - \beta^2}{(1 + \beta\beta_o)^2} \right] \quad (10)$$

If t_i and t_o are clock readings and x_i and x_o are coordinates in frame i and frame o, then:

$$t_i = \frac{t_o + x_o\beta/c}{\sqrt{1 - \beta^2}} \quad (11)$$

Knowing that $c\beta_o = dx_o/dt_o$, differentiating (11) gives:

$$\frac{dt_i}{dt_o} = \frac{1 + \beta\beta_o}{\sqrt{1 - \beta^2}} \quad (12)$$

Since $\alpha = \frac{d\beta}{dt}$, combining (10) and (12) gives:

$$\alpha_i = \frac{\alpha_o (1 - \beta^2)^{3/2}}{(1 + \beta\beta_o)^3} \quad (13a)$$

$$\alpha = \alpha_o (1 - \beta^2)^{3/2} \quad (13b)$$

Equation (13a) is the general result for the acceleration seen by frame i. However, the interesting case is when the object is stationary in frame o ($\beta_o = 0$). This is (13b). The subscript 'i' will now be dropped for quantities measured in frame i and the frame i acceleration is denoted as α to indicate this is a general observed acceleration of any object traveling with speed β .

When an object undergoes a constant proper acceleration α_o , (13b) can now be used to generate general expressions for quantities seen by frame i. The clock reading in frame i will be t . Assuming the object starts accelerating in frame i from coordinate zero at time zero, velocity as a function of time is:

$$\beta = \frac{\alpha_o t}{\sqrt{1 + (\alpha_o t)^2}} \quad (14a)$$

$$\alpha_o t = \frac{\beta}{\sqrt{1 - \beta^2}} \quad (14b)$$

