The principles demonstrated for the orthogonal gravitational field in the article *Special Relativity and Gravity* can also be applied to circular gravitational fields.

CYLINDRICAL GRAVITATIONAL FIELDS

One type of circular gravitational field would be the cylindrically shaped field surrounding the pole mass that generates the DEOGF in the x-Q plane shown in the article *Special Relativity and Gravity*, Figure 1. Referring to Figure 1, this cylindrical gravitational field would exist in a plane perpendicular to the x-axis of the pole. See Figure 5.

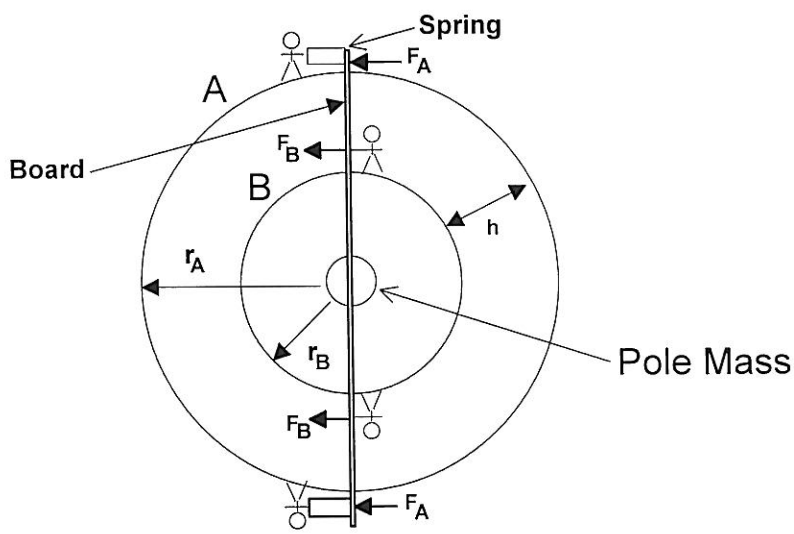

Figure 5

Reference frames A and B are cylindrical surfaces at constant radial distances r_A and r_B from the pole mass. This example is just a different view the DEOGF of Figure 1, so many characteristics of this circular gravitational field have already been calculated. The Acceleration Law for this circular gravitational geometry is a modification of (4) in which the coordinate h is the distance above the baseline radius $d = r_B$ where the gravitational acceleration is g_o.

$$g_h = \frac{g_o}{1 + \frac{g_o h}{c}} \qquad (16)$$

One item that has not been previously calculated is the relationship of forces in the circumferential direction between reference frames. In order to make this calculation, reference frames A and B can be constructed to be fixed to the infinite pole mass with supports that prevent movement of these surfaces.

Now assume a board spans the distance between frames A and B. The board passes through a small hole in the infinite pole mass to access the mirror image experiment on the opposite side. As two frame B observers push on the board in frame B, the board will push on springs in frame A. The observers pushing on the board each apply force F_B and the observers holding the springs in frame A each feel force F_A. The springs are compressed and clamped in the compressed condition. Each spring contains energy E when clamped in the compressed condition in frame A. Each spring is lowered to frame B by observer B (with a pole) where it still contains energy E. The work obtained by frame B as a spring is lowered is W_{AB}.

$$W_{AB} = \left(\frac{E}{c^2}\right) g_B c h \tag{17}$$

The energy contained in the spring is $E = \frac{1}{2} F_A dy$, with dy being the small displacement of the board in the circumferential direction in Frame A. The energy expended initially by observer B is $\frac{1}{2} F_B dy$. With the spring energy moved down to frame B, the Law of Conservation of Energy gives:

$$\tfrac{1}{2} F_B dy = \tfrac{1}{2} F_A dy + W_{AB} = E + W_{AB}$$

$$\frac{F_B}{F_A} = \left(1 + \frac{W_{AB}}{E}\right) = \left(1 + \frac{g_B h}{c}\right) \tag{18}$$

This is the same result for the circumferential direction as (14a) is for the x-axis direction.

SPHERICAL GRAVITATIONAL FIELDS

A spherical gravitational field surrounds a point mass or approximates the shape of a gravitational field surrounding a spherical object such as a planet. The governing relationship of this shape of field will be given by (1).

The creation of an Acceleration Law for a spherical gravitational field will follow the same logic as for the gravitational fields of other shapes presented previously. First, a calculation must be made to find the pole work W_{AB} obtained when a mass m undergoes a change in position from a reference frame A at radius r_A to a reference frame B at radius r_B. The large mass M is providing the gravitational field and the smaller mass m is changing radial position within that field.

$$W_{AB} = \int_A^B - \frac{GMm}{r^2} dr$$

$$W_{AB} = GMm\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \tag{19}$$

If the smaller mass is actually an amount of energy $m = E/c^2$:

$$\frac{W_{AB}}{E} = \frac{GM}{c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \tag{20}$$

Equations (19) and (20) will be used in a new experiment similar to the one shown in Figure 5, only now the infinite pole mass will be replaced by a planet. See Figure 6.

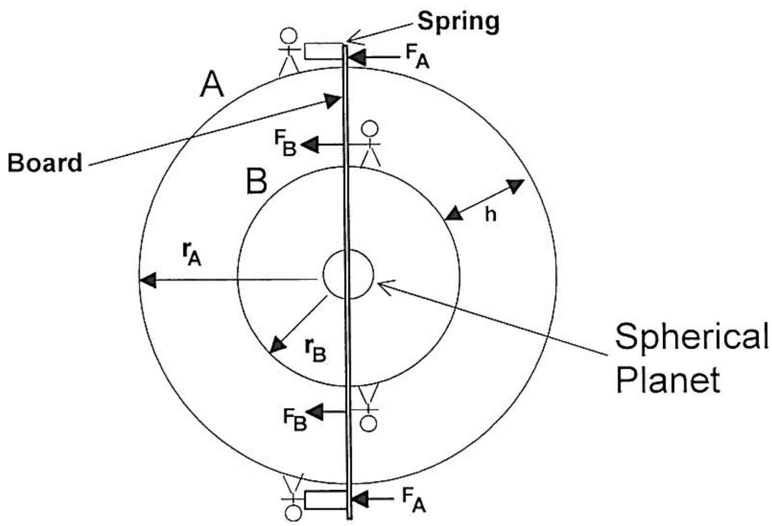

Figure 6

The board has a hole in it that passes around the planet. The springs will again be compressed so that each contains energy E. Each spring will be clamped in position and lowered to reference frame B. The energy obtained when a spring is lowered is W_{AB} as given by (19). When the spring has relocated to frame B, its potential energy in frame B is still E.

The energy contained in the spring is $E = \frac{1}{2}F_A dy$. The energy expended initially by observer B is $\frac{1}{2}F_B dy$. With the spring energy moved down to frame B, the Law of Conservation of Energy gives:

$$\tfrac{1}{2}F_B dy = \tfrac{1}{2}F_A dy + W_{AB} = E + W_{AB}$$

$$\frac{F_B}{F_A} = (1 + \frac{W_{AB}}{E})$$

$$\frac{F_B}{F_A} = 1 + \frac{GM}{c^2}(\frac{1}{r_B} - \frac{1}{r_A}) \qquad (21)$$

This solution for forces can also be expressed as:

$$F_A = mg_A c \qquad\qquad F_B = \frac{GMm}{r_B^2} = mg_B c$$

$$\frac{g_B}{g_A} = 1 + \frac{GM}{c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) = 1 + \frac{g_B h}{c}\left(\frac{r_B}{r_A}\right) \qquad (22)$$

Equation (22) can be compared to (16) to see how the structure of the Acceleration Law changes when the gravitational field shape goes from cylindrical to spherical. Equation (22) can also be compared to (21) from *The Acceleration Law* to see how gravitational fields from point masses differ from acceleration patterns of dynamic experiments.

The expression for time rates in the two reference frames is found using the Law of Conservation of Momentum. The experiment of Figure 6 is modified as shown in Figure 7.

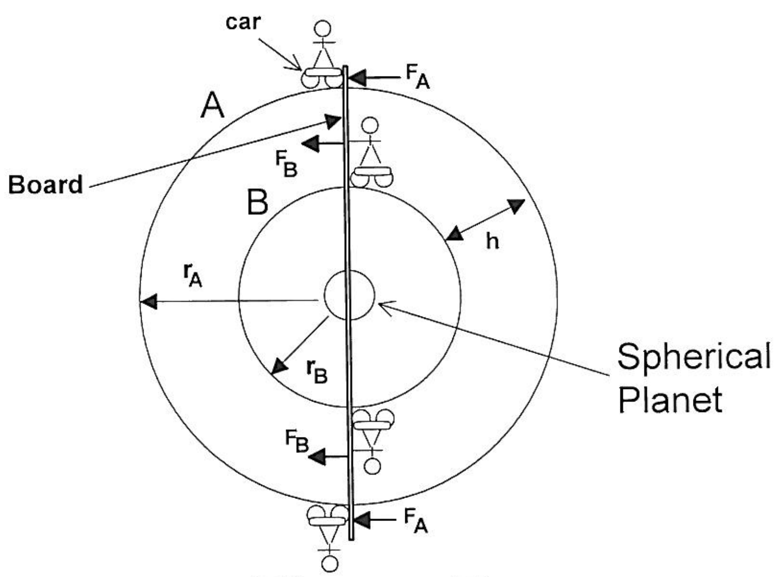

Figure 7

Reference frames A and B will be constructed to be similar to railroad tracks. These tracks hold identical cars in fixed radial positions. The only movement of the cars is in the circumferential direction along the tracks. The cars also resist motion in any other direction regardless of the force or torque applied to them.

Now assume a board is fixed to the car in frame A and spans distance h between reference frames. As the frame B observers standing on their cars push on the board, they will each apply a force F_B in the directions shown. They apply forces to their cars in the opposite direction. The board will push on the cars in frame A with a force F_A as given by (21). If clocks in both frames read zero as the forces are applied, then the clock in frame A will read t_A and the clock in frame B will read t_B when the force application ends. The change in momentums of the cars in frame B and frame A must be equal.

$$F_A t_A = F_B t_B$$

$$\frac{t_A}{t_B} = 1 + \frac{GM}{c^2}\left(\frac{1}{r_B} - \frac{1}{r_A}\right) \tag{23}$$

Equation (23) gives the relative clock readings (time rates) for frame A and frame B.

COMPARISON TO SCHWARZSCHILD AND EXPERIMENTAL EVIDENCE

The Schwarzschild solution to the General Theory of Relativity gives the following equation for time dilation in a spherical, non rotating gravitational field.

$$t_r = t_\infty \sqrt{1 - \frac{2GM}{rc^2}} \tag{24}$$

r - radius from center of mass M
t_r - local clock reading between two events at radius r
t_∞ - clock reading between same two events seen from $r = \infty$

Equation (24) is considered to be confirmed by experimental evidence gathered over many years. Equations (23) and (24) appear to be significantly different. However, a comparison of calculated values for these two equations will be made. This test case will use commonly accepted values for gravitational time dilation for Global Positioning System satellites orbiting the earth. The time accuracy required of these satellites makes gravitational time dilation significant for accurate position coordinates to be achieved. The Schwarzschild solution (24) gives:

GM - 3.9912 x 10^{14} m^3/sec^2
$r_B - 6,360,000$ m (radius of earth's surface)
$r_A - 26,360,000$ m (radius of satelite orbit)
$c - 300,000,000$ m/s
$t_{earth} = 86400$ seconds/day
$t_{satelite} = 86400.0000457091$ seconds/day (25)

The same values applied to (23) give:

$t_{satelite} = 86400.0000457091$ seconds/day (26)

The accepted value of gravitational time dilation for GPS satellites is 45.7 microseconds per day. This number may change slightly depending on the values of inputs used or if the assumption that the earth is not exactly a sphere is considered. Repeating this comparison of (23) and (24) for various other values of inputs shows that the two equations are in agreement for every experimental test condition used to verify time dilation predicted by the General Theory of Relativity. The only area that the two equations give separate results is when values of r_B approach the Schwarzschild radius, $r_B = 2GM/c^2$. At this radius, (24) gives a singularity and (23) does not. Equation (23) gives a singularity at $r_B = 0$ (and mass M is assumed to be concentrated at that one specific point).

Other experimental evidence supporting the effect of gravity on time per the General Theory, such as gravitational red shifting of light frequencies from dense stars, also supports this new theory, which will be referred to as the Special Theory of Gravity.

SUMMARY

The comparison of (23) and (24) is evidence that the acceleration and gravitational theory based on the Special Theory of Relativity is valid. Using the Special Theory of Relativity for gravitational fields gives some additional information that the General Theory does not - specifically, more information on forces is included with the Special Theory of Gravitation. Other information, such as local applications of momentum and energy laws, is also easier to compute. Although the General Theory of Relativity may be considered the more sophisticated theory, the Special Theory of Gravity will have advantages in analyzing local experiments where observer interaction between reference frames is required. It is also easier to use, especially when new shapes of gravitational field are being investigated. The Special Theory of Gravity also relates directly to dynamic experiments and can present exact parallels or differences between gravitational and dynamic experiments.