In the article *Force and Geometry,* (23a) is a transformation for a force applied in the direction of movement of an inertial reference frame relative to another stationary reference frame. That discussion was only concerned with the case where mass m was instantaneously at rest in the moving inertial reference frame A. The case where there is an x-direction velocity to mass m in moving frame A is also of interest.

The General Transformation

In Figure 15, the experiment is similar to the experiment of Figure 7 except that there is no y-direction force on mass m. Frame B has an observer who is applying a constant force F_{XB} to mass m. In this moment, mass m is moving with an instantaneous x-direction velocity β_{XA} relative to frame A and β_{XB} relative to frame B.

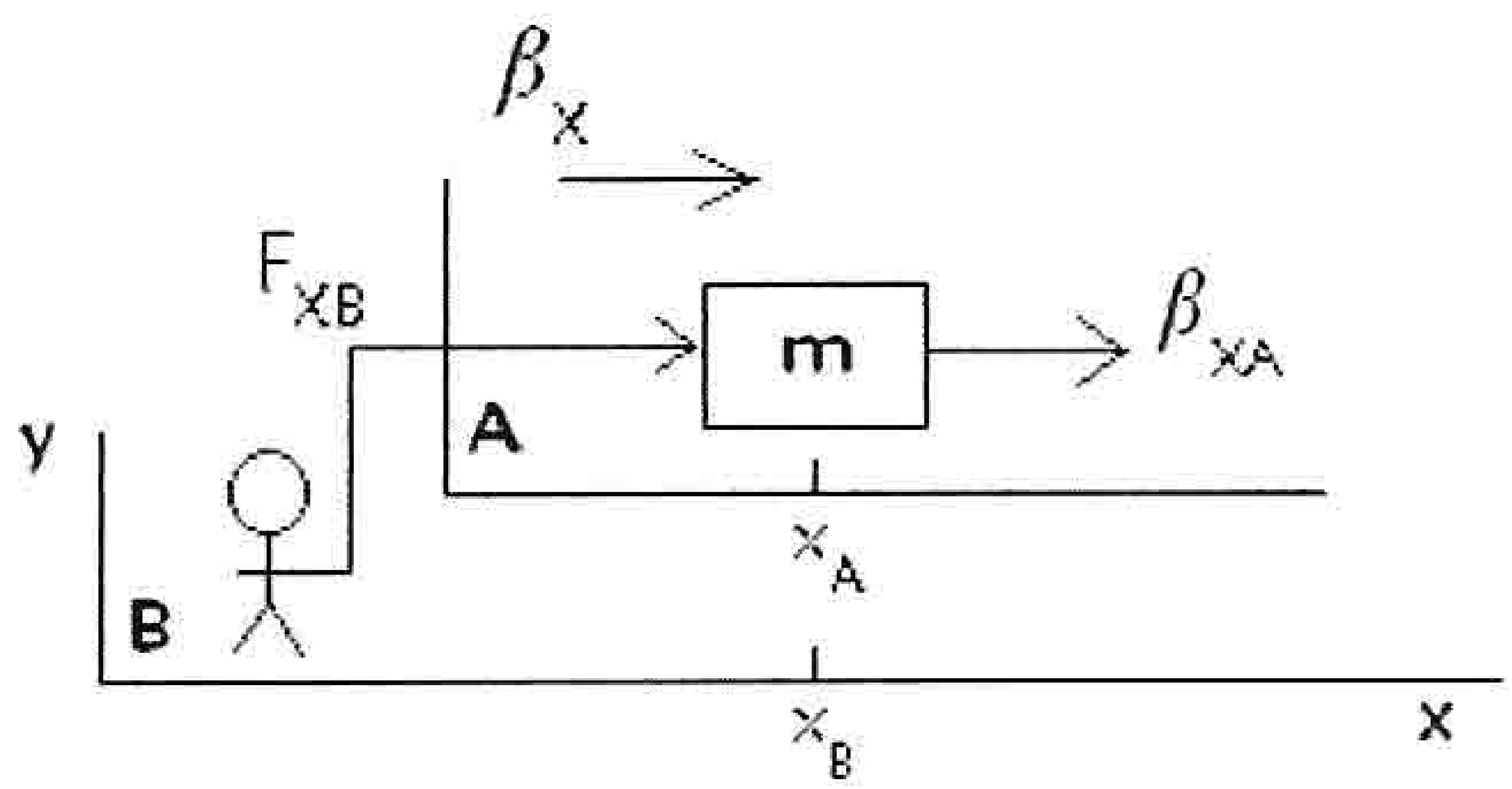

Figure 15. Another simple force experiment.

No momentum change of frame B occurs during this experiment because a mirror image experiment with exactly opposite forces is run simultaneously by frame B, but is not shown. Both frames see mass m accelerate and both apply (20). F_{XA} is the force that frame A would apply to mass m to get the same observed acceleration as F_{XB}.

$$F_{XA} = \frac{mcA_{XA}}{\left(1-\beta_{XA}^2\right)^{3/2}} \tag{47a}$$

$$F_{XB} = \frac{mcA_{XB}}{\left(1-\beta_{XB}^2\right)^{3/2}} \tag{47b}$$

The velocities in (47) can be related by using (14):

$$\beta_{XB} = \frac{\beta_{XA} + \beta_X}{1 + \beta_X \beta_{XA}} \tag{48}$$

The accelerations in (47) can be related by using (18):

$$A_{XB} = A_{XA} \frac{\left(1 - \beta_X^2\right)^{3/2}}{\left(1 + \beta_X \beta_{XB}\right)^3} \tag{49}$$

Inserting (48) and (49) into (47b) gives:

$$F_{XB} = \frac{mcA_{XA}}{\left(1 - \beta_{XA}^2\right)^{3/2}} = F_{XA} \tag{50}$$

Equation (50) has a meaning that is slightly different than (23a). Equation (23a) says that the force that mass m "feels" (measured in a reference frame in which it is instantaneously stationary) is the same as the force applied to it by frame B. Equation (47) says that any reference frame with an x-direction velocity will interpret the motion of mass m as being accelerated by a force equal to F_{XB}. In other words, F_{XB} is a constant to all reference frames traveling in the applied direction.

Time and Force Application

In Figure 16, the observer in frame B is applying a constant force F_{XB} to mass m for a time period t_B. During this time period, mass m accelerates to coordinate x_B. After that point, the observer stops pushing and mass m travels at constant velocity β_X. A mirror image experiment (not shown) is run in the opposite direction so that frame B does not have a change in momentum during the experiment.

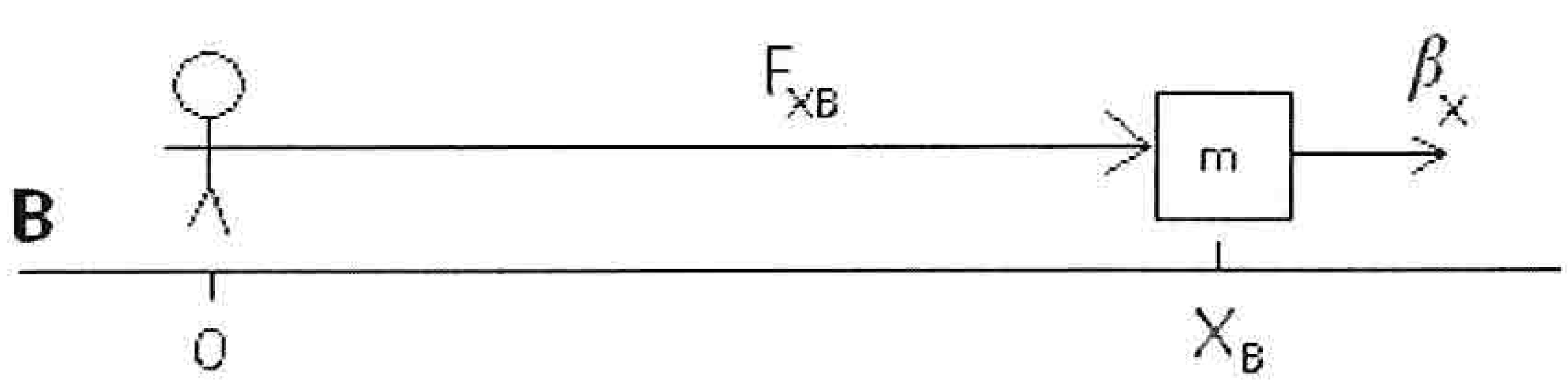

Figure 16. A simple experiment in frame B.

Mass m starts the experiment at frame B coordinate zero (where the observer is standing).

Mass m must feel force F_{XB} stop instantly at x_B. However, if the observer stops applying force F_{XB} as mass m reaches x_B, mass m will continue to accelerate until the "signal" of the force application travels distance x_B. Mass m will continue to accelerate for a time period of x_B / c. This is not what the observer wants.

This conflict results because the mechanism of force application is not clearly defined. For example, the mechanism could be defined as shown in Figure 17.

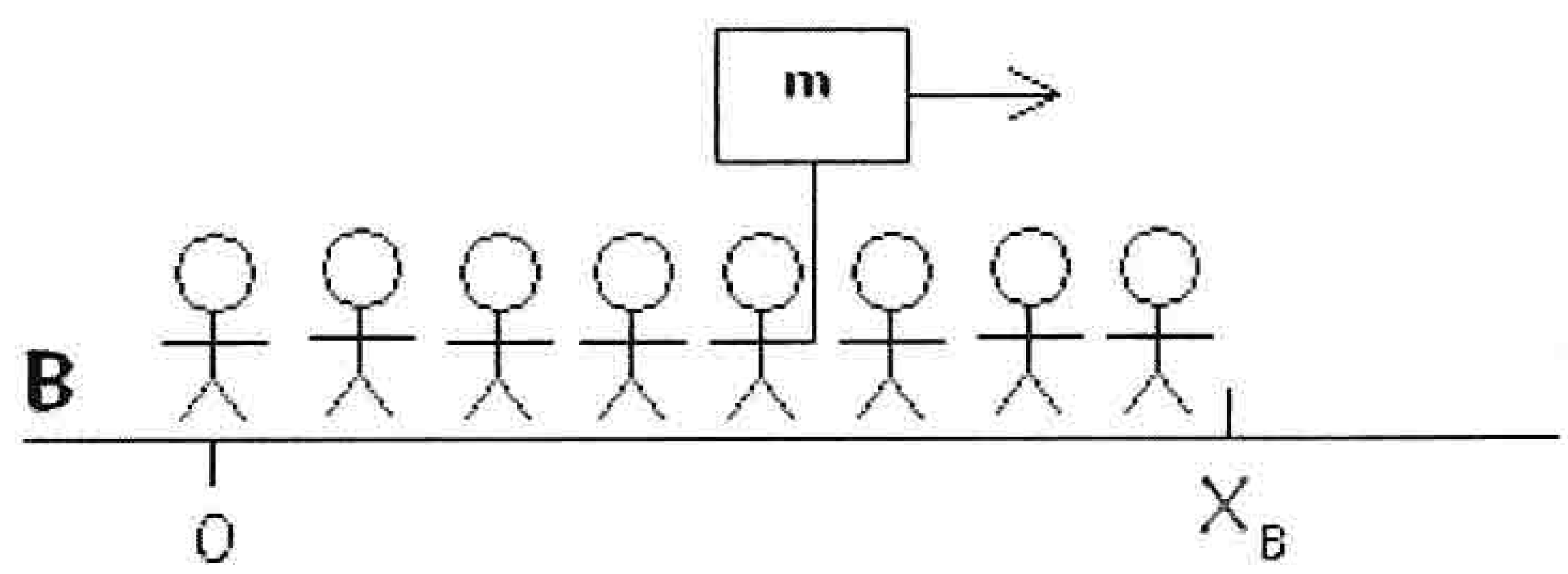

Figure 17. A row of observers pass mass m along.

In Figure 17, mass m is shown at an intermediate position as it is accelerated to position x_B. Each observer is only an incremental distance along the x-axis from the next. Each applies force F_{XB} in turn to mass m as it passes. When the application of force stops at position x_B, the mass instantly stops accelerating. There is no "signal" delay.

Another acceleration mechanism is shown in Figure 18, where another line of observers pushes mass m, but this time they use a pole. The length of the pole is x_B.

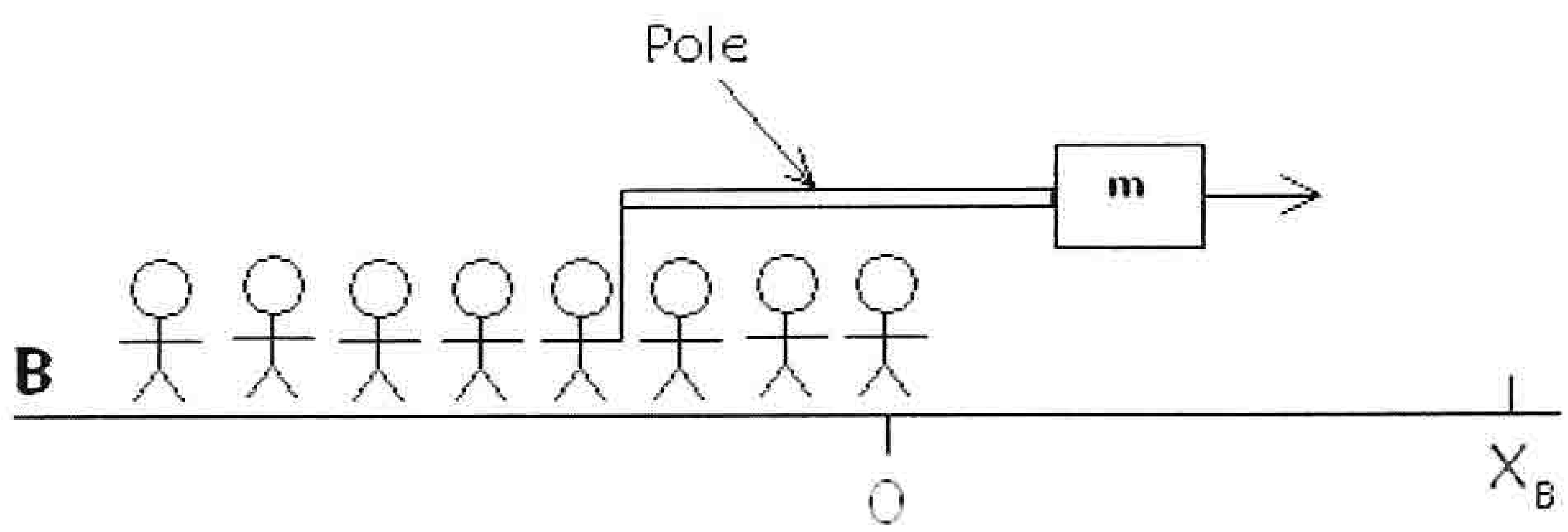

Figure 18. Another mechanism to accelerate mass m.

In Figure 18, as the end of the pole gets up to position zero, frame B stops applying the force F_{XB}. However, at this point mass m is not at coordinate x_B and has not finished accelerating. The pole now has a velocity that is almost β_X and has undergone length contraction. The clock at the end of the pole touching mass m is also "showing an earlier time" than the clock at the end of the pole touched by the observers. Therefore, even though the observers have stopped pushing on their end of the pole, mass m is still being accelerated by the other end of the pole. The force F_{XB} pushing on mass m at this last moment is now a "mysterious force" with no obvious source (according to frame B).

The mysterious force demonstrated in the article *Force and Geometry* was in the x-direction. The mysterious force demonstrated above is in the "time" direction. This reinforces the linkage between force and (four dimensional) geometry. There is also a type of mysterious force at the beginning of the experiment. When the first observer starts accelerating the pole, mass m doesn't move until the "signal" of force travels up the pole to mass m. This initial force, which acts before mass m moves, could also be called mysterious.

The view of what happens to the length of an accelerating pole has been explained in detail in the article *The Acceleration Law*. All of that discussion applies to this case, even though there is now a force on each end of the pole and this results in energy passing through the pole. If the pole has infinite stiffness in this ideal universe, there is no reason to believe the geometry of an accelerating pole will change just because it has a static force on each end.

A third type of acceleration mechanism is shown in Figure 19, where a single frame B observer uses a pole to accelerate mass m. The single observer accelerates the pole using a hand-over-hand motion. Therefore, one observer inputs all the acceleration energy for the experiment and the length of the pole (between the observer and mass m) is constantly changing. This is a challenging mathematical problem to solve for intermediate positions of acceleration, but the final position of the pole is nearly the same as illustrated in Figure 18. Therefore, what happens to the pole and mass m is similar to the explanation of Figure 18. This method could simulate a spring placed between frame B and mass m.

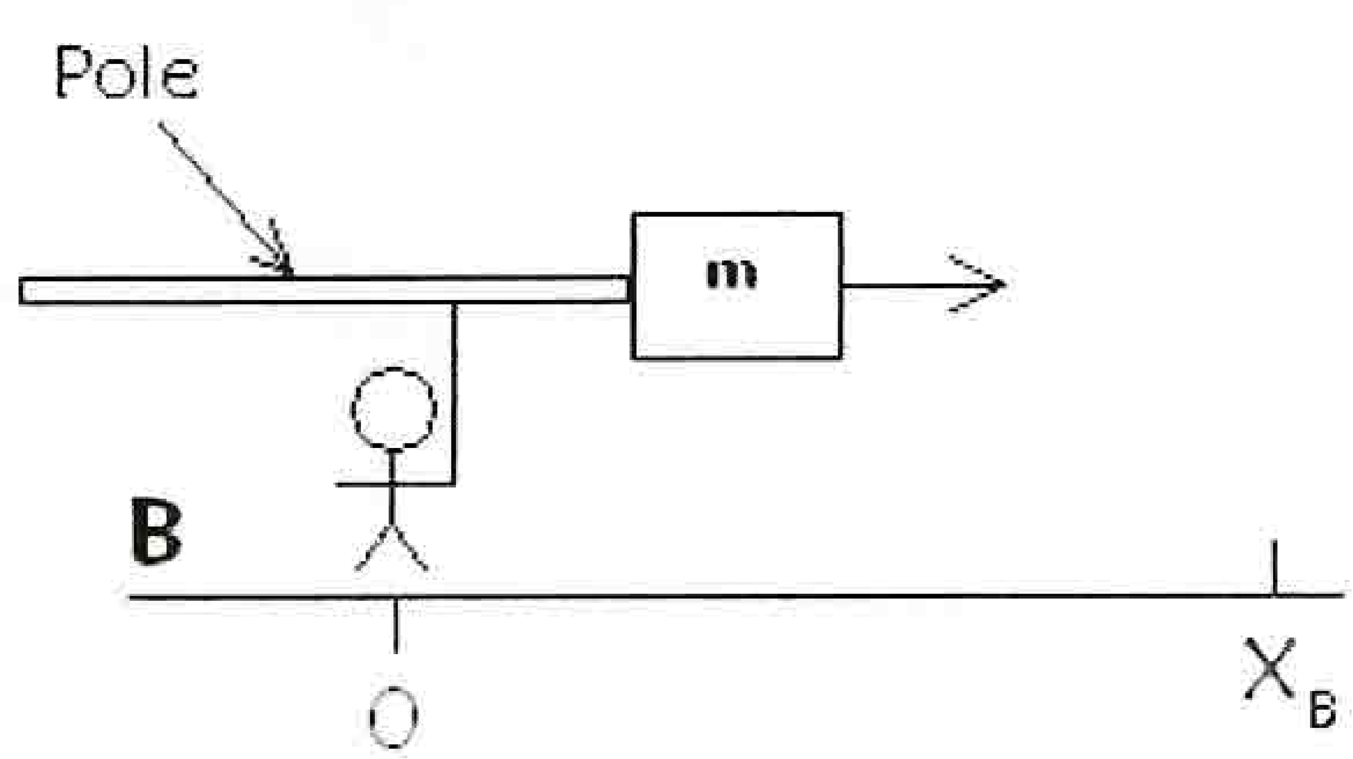

Figure 19. A single observer accelerates the pole hand-over-hand.

Failure of Force Simultaneity at a Distance

The existence of mysterious forces seems to be related to a linkage between force and geometry within experiments. "Failure of simultaneity at a distance" is affecting the force that is applied between the two different reference frames. But the force application method of Figure 17 appears to eliminate any simultaneity effects and would provide reason to believe that force and geometry are not always linked.

To decide if force and geometry are linked using the application method of Figure 17, assume that this method is used in the experiment of Figure 15. For additional clarification, it is now stated that mass m receives the force application starting at clock readings of zero in both reference frames and at coordinate readings of zero in both reference frames (the frame origins coincide at the start of the experiment). Assume that the force application stops when mass m gets to coordinates x_A and x_B (which are opposite each other in their respective frames). Mass m now continues onward at constant velocity β_{XA}.

Now, consider Figure 20, which is the experiment as seen by frame A at this point.

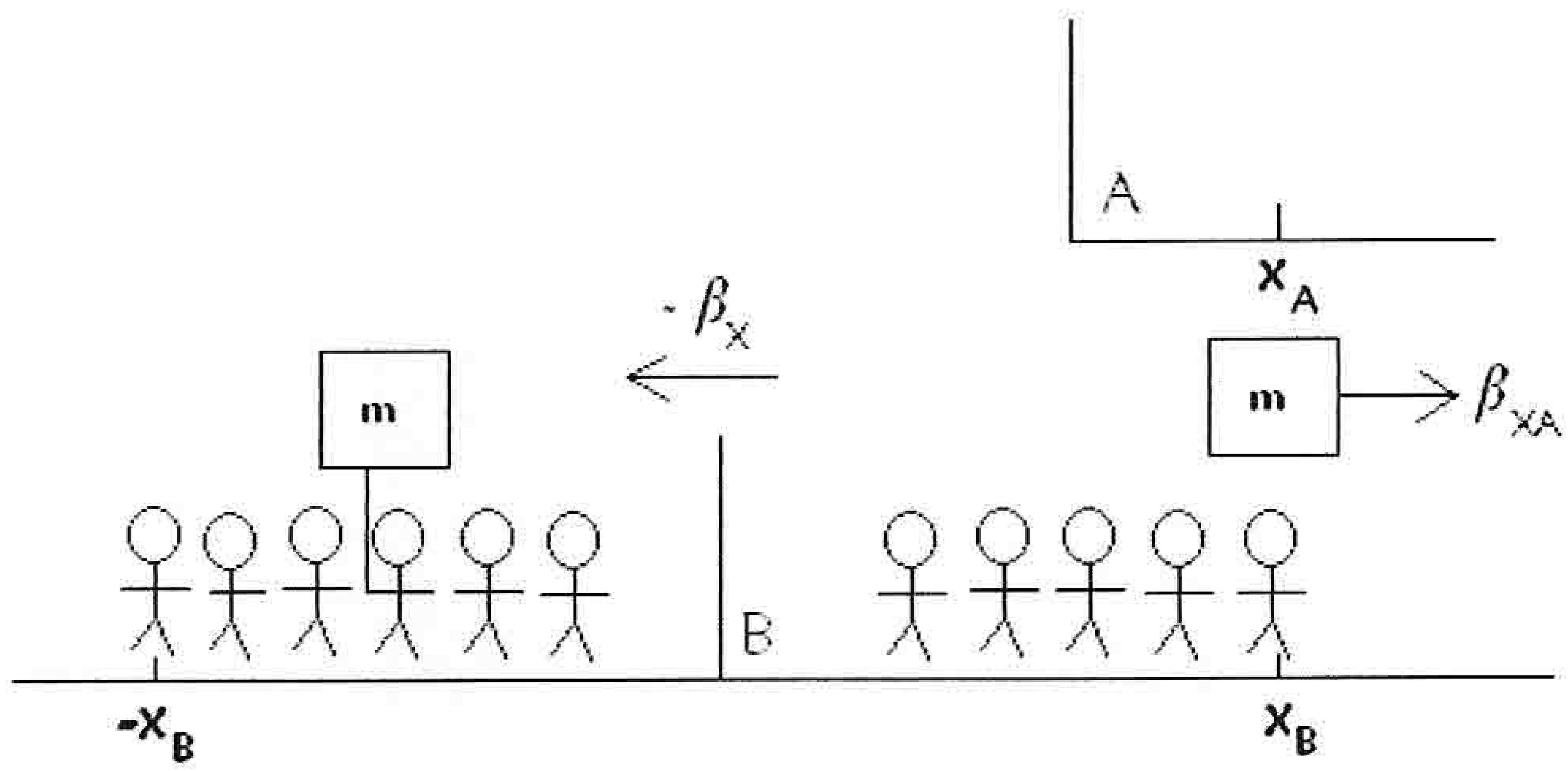

Figure 20. Frame A view of the experiment of Figure 15.

Figure 20 shows the experiment just as mass m reaches coordinate x_B. Frame A sees frame B passing by with velocity $-\beta_X$. Mass m has just stopped accelerating. But, frame A also sees a second mass m still being accelerated by the frame B observers. This mass m is part of the "mirror image experiment with exactly opposite forces is run simultaneously by frame B" that was not shown in Figure 15. The acceleration of this second mass is required to avoid a momentum change of frame B during the experiment.

Although both masses reach their respective coordinates x_B and $-x_B$ simultaneously when viewed by frame B, frame A does not see this occur due to failure of simultaneity at a distance. Frame A sees the second mass reach coordinate $-x_B$ and time t_{A2}. The extra time between t_{A2} and t_A is Δt_A and:

$$\Delta t_A = t_{A2} - t_A = \frac{\frac{2x_B \beta_X}{c}}{\sqrt{1-\beta_X^2}} \tag{51}$$

Δt_A is the time period when the second mass m is being accelerated without a mirror image force to keep frame B from changing momentum. The force on the second mass m is a mysterious force during this time period. Time period Δt_A does not result from a "signal" delay running down a pole to the second mass m because there is no pole. Δt_A is a direct result of failure of simultaneity at a distance within the geometry of the experiment (a "signal" delay in the dimension of the reference frame).

Conclusions

Forces in the direction of motion are interpreted to be the same value by all reference frames traveling in that direction. However, the time dimension is affected by force application in this situation. Specifically, failure of force simultaneity at a distance generates mysterious forces in any experiment.

The articles *The Real Ladder Paradox* and *The Acceleration Law* showed that energy and geometry are linked at some fundamental level. Since energy is a combination of force and geometry, these articles are essentially saying that force and geometry are linked at a fundamental level. This article and the article *Force and Geometry* explain some of the details of that linkage.

The linkage of force and "failure of simultaneity at a distance" reinforces the idea that "force enhancement" is a principle of relativity and not just a coincidental occurrence in some specific experiment. Failure of force simultaneity at a distance is not a property of some particular component of an isolated experiment (e.g. a pole). It is a part of the geometry of the dimensional space in the experiment. It describes force events in Special Relativity in the same way as the other three principles of relativity describe dimensional events. It requires force to be included in all relativistic experiments, even though relativity was initially conceived as an inertial physics where force played no role.