Equations present a mechanism to evaluate thought experiments, but often do not give a visual picture of the concepts being presented. Two specific cases will be presented to show acceleration in Special Relativity.

Case 1: rod length $L = 10^6$ meter, $\alpha_B = 0.1$, final velocity $\beta = 0.999$

Case 2: rod length $L = 10^6$ meter, $\alpha_B = 10$, final velocity $\beta = 0.5$

Rod Length During Acceleration

Figure 17 shows a rod (connecting object A and object B) accelerating (as a reference frame) relative to stationary reference frame i. Object B starts accelerating from coordinate zero. The length of the stationary rod is L. Equation (21) defines α_A and the acceleration observed by frame i is given by (13b). At any point in the acceleration, the coordinate position x_i of either object is given by (15).

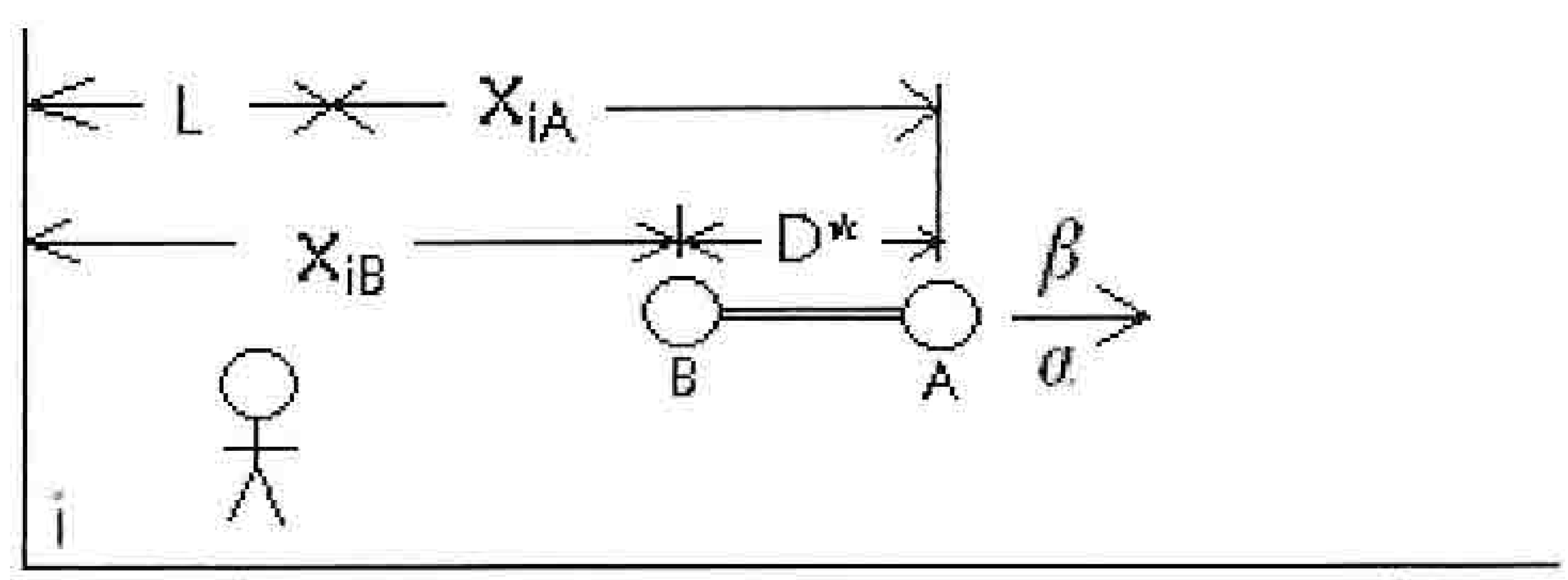

Figure 17. What does accelerating rod AB look like?

If the instantaneous velocity of the object is defined to be the velocity of object B, then the length of the object during the acceleration is D^*. Starting with (14b), the frame i time required to reach $\beta = 0.5$ is t.

Case 1 $$t = \frac{0.5/0.1}{\sqrt{1-0.5^2}} = 5.7735 \text{ sec} \qquad \alpha_A = \frac{0.1}{1+\frac{0.1(10^6)}{3\times 10^8}} = .0999667 \text{ sec}^{-1}$$

$$D^* = (x_{iA} + L) - x_{iB} = (4.6397\times 10^8 + 10^6) - 4.641\times 10^8 = 866061\ m$$

$$D = L\sqrt{1-\beta^2} = 866025\ m$$

$$D^*/D = 1.0000417 \qquad (48)$$

Case 2 $t = \dfrac{0.5/10}{\sqrt{1-0.5^2}} = 0.057735 \text{ sec}$ $\alpha_A = \dfrac{10}{1+\dfrac{10(10^6)}{3\times 10^8}} = 9.6774\,\text{sec}^{-1}$

$$D^* = (x_{iA} + L) - x_{iB} = (4.5106\times 10^6 + 10^6) - 4.641\times 10^6 = 869546\ m$$

$$D = L\sqrt{1-\beta^2} = 866025\ m$$

$$D^*/D = 1.0041 \qquad (49)$$

D is the length of the rod AB if the entire rod were traveling inertially at speed β. D is not equal to D^*. This is because frame i does not see the velocity along the accelerating rod to be uniform down the length of the rod. For example, the velocity of object A at time t for this example is found from (14a) and is:

Case 1 $\beta_A = \dfrac{\alpha_A t}{\sqrt{1+(\alpha_A t)^2}} = 0.499875$ Case 2 $\beta_A = 0.48776$ (50)

If the length of the rod were determined by the velocity of object A at this instant then:

Case 1 $D_A = 866098\ m$ $D^*/D_A = 0.99996$

Case 2 $D_A = 872980\ m$ $D^*/D_A = 0.99607$ (51)

For Case 1, with object B at velocity $\beta = 0.999$ (object B defines the rod velocity):

$t = 223.439$ sec $D^* = 44718\ m$ $D = 44710\ m$ $D^*/D = 1.00017$ (52)

If the acceleration of the rod stops when it reaches velocity β, frame i sees a gradual cessation of the acceleration progress down the length of the rod from object B to object A, after which the entire rod is traveling at a constant velocity.

Clock Readings During Acceleration

Assume identical clocks are place along the frame i axis and on objects A and B. All clocks read zero as the acceleration begins. Frame i will see these clocks read according to (16) during the acceleration. At the point where the object B velocity is $\beta = 0.5$:

Case 1 Object B clock reading $t_B = \dfrac{1}{\alpha_B}\ln\left[\alpha_B t + \sqrt{1+(\alpha_B t)^2}\right] = 5.49306$ sec

Object A clock reading $t_A = 5.49323$ sec

Case 2 Object B clock reading $t_B = 0.0549306$ sec

Object A clock reading $t_A = 0.0550883$ sec (53)

The object A clock is later than the object B clock (its velocity is always less than object B), and both object clocks are less than the frame i clock reading.

When the rod reaches its final velocity, the cessation of acceleration proceeds down the length of the rod from object B to object A. During this process, part of the rod will be traveling at a constant velocity and part will still be accelerating. Clock readings would have to be calculated using a combination of inertial and acceleration equations. For Case 2, object A will achieve velocity $\beta = 0.5$ at the frame i clock reading given by:

$$t_{iA} = \frac{\beta / \alpha_A}{\sqrt{1-\beta^2}} = 0.05966 \text{ sec} \tag{54}$$

The Case 2 object clock readings at the instant where object A gets to $\beta = 0.5$ are:

Object A $$t_{A\beta} = \frac{1}{\alpha_A} \ln\left[\alpha_A t_{iA} + \sqrt{1+(\alpha_A t_{iA})^2}\right] = 0.0567616 \text{ sec}$$

Object B $$t_{BA\beta} = (t_{iA} - t)\sqrt{1-\beta^2} + t_B = 0.056597 \text{ sec} \tag{55}$$

Frame i sees the object A clock read later than the object B clock when the entire rod is finally traveling at velocity $\beta = 0.5$.

Acceleration Failure of Simultaneity at a Distance

The object A clock readings noted in (53) and (55) have been affected significantly by the acceleration. An inertial frame ii traveling at $\beta = 0.5$ would see objects A and B appear to simultaneously stop moving as the rod achieves this velocity. The times appearing on the object clocks at this instant would be $t_{A\beta}$ and t_B. These clocks would no longer be synchronized, although they had been synchronized when the acceleration started (when they all read zero). Observers along the accelerating rod would also agree that the clocks were no longer synchronized during the acceleration.

A comparison can be made between these clock readings, using (19) and (21), taking Case 2 as an example.

$$\frac{t_{A\beta}}{t_B} = 1.03333 = \frac{\alpha_B}{\alpha_A} = 1 + \frac{\alpha_B L}{c} \tag{56}$$

It is possible to define an "acceleration failure of simultaneity at a distance" as:

$$\Delta t_{AB} = t_{A\beta} - t_B = t_B\left(\frac{\alpha_B L}{c}\right) = \frac{L}{c}\ln\left[\alpha_B t + \sqrt{1+(\alpha_B t)^2}\right]$$

$$\Delta t_{AB} = \frac{L}{c}\ln\left[\sqrt{\frac{1+\beta}{1-\beta}}\right] \tag{57}$$

As viewed by the stationary frame i, an accelerating object clock at the 'front' will read later than the clock in the 'back'. Note that Δt_{AB} is not dependent on the acceleration value. Any acceleration to velocity β will result in the same Δt_{AB}.

Summary

An object of length accelerating to a final velocity β undergoes a combination of effects during the acceleration process. At first, purely accelerative effects are present in the rod's length and time characteristics. As the rod reaches speed β, the constant speed condition sweeps up the length of the rod starting at the 'back' and proceeding to the 'front'. Length contraction follows this sweep as the rod converts from an acceleration based length contraction to the more familiar inertial length contraction. Clock readings also convert from an acceleration based rate change to the inertial rate in a pattern following this sweep forward.