In the article *A Momentum Paradox*, a simple device was presented. This device creates a paradox because Special Relativity does not provide techniques which can analyze the operation of many simple mechanisms. In this article, new experiments similar to the one in *A Momentum Paradox* will be presented. The momentums of these experiments will be calculated to see if these experiments obey the Law of Conservation of Momentum.

Moving Mass Experiment

Figure 3 shows the first thought experiment.

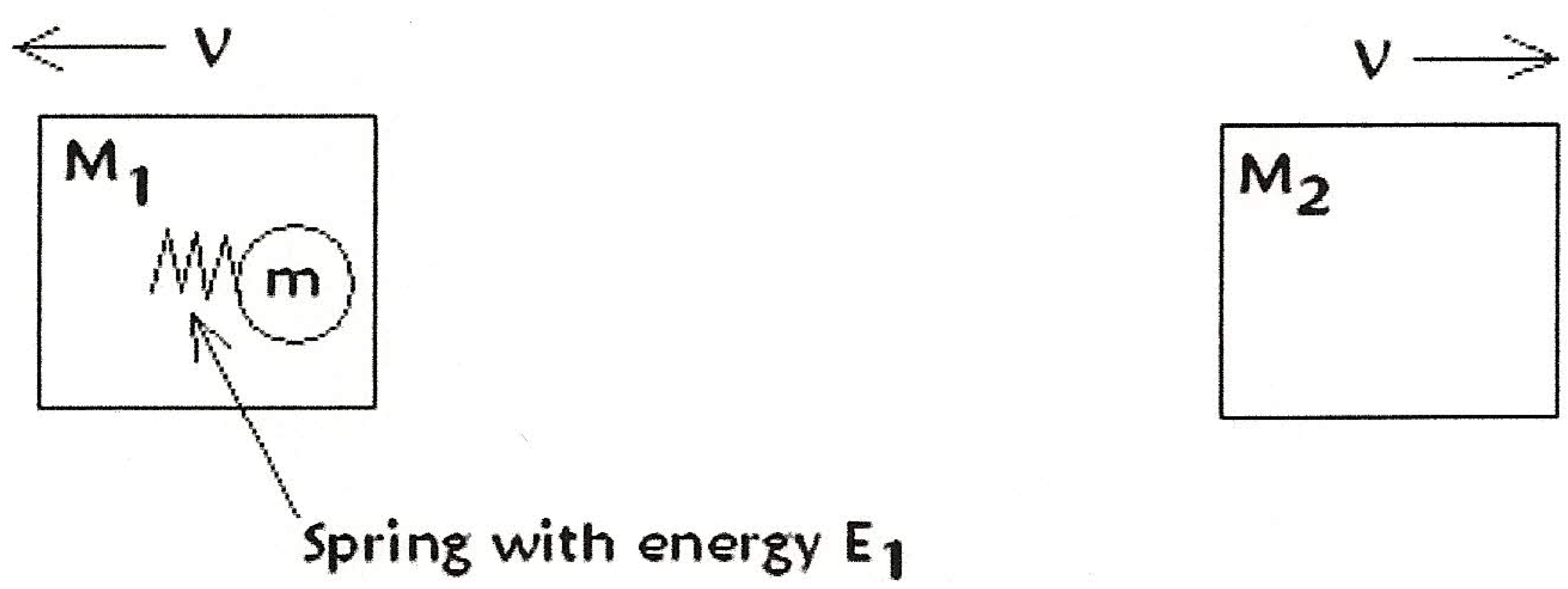

Figure 3. Starting position of Moving Mass experiment.

In Figure 3, two masses are moving with the velocities shown relative to our stationary inertial reference frame. The left mass M_1 contains a smaller mass *m* and a spring that is compressed and contains energy E_1.

$$M_2 = M_1 + m + E_1 / c^2 \tag{4}$$

Under the condition (4), these two masses will have equal momentums relative to our observing reference frame (although these momentums are opposite in direction).

When the experiment starts, mass *m* will be sent by the spring toward mass M_2. If it's velocity is sufficient, it will run into M_2 and will compress a spring inside of M_2 (not shown). The energy in this spring will be E_2 after compression.

The first part of the experiment will be analyzed from a reference frame that is traveling to the left at speed *V*. See Figure 4, where all of the spring energy has been expended and mass *m* has a velocity to the right.

Figure 4. Left reference frame view after mass *m* has been pushed by the spring.

It must be assumed that the Law of Conservation of Momentum applies for the event depicted in Figure 4. The spring energy has been converted into kinetic energy of the two masses and does not contribute to the momentum calculation.

$$\frac{M_1 V_{1L}}{\sqrt{1-\frac{V_{1L}^2}{c^2}}} = \frac{m v_L}{\sqrt{1-\frac{v_L^2}{c^2}}}$$

$$V_{1L} = \frac{-c}{\sqrt{1+\frac{M_1^2}{m^2}\left(\frac{c^2}{v_L^2}-1\right)}} \tag{5}$$

In (5), V_{1L} has been made negative to indicate its direction. Any velocity v_L can be chosen for *m* and the corresponding velocity for M_1 can be found from (5). To observe the event where *m* runs into mass M_2, a reference frame traveling at speed *V* to the right is chosen. See Figure 5.

Figure 5. Right reference frame view of mass *m* before it collides with M_2.

From the right frame, M_2 is initially stationary and mass m has a different velocity from that observed by the left frame. This velocity is found from the velocity addition formula (see (9) in the article *The Acceleration Law*). But before v_R can be found, the velocity of the left frame as seen by the right frame must be found. This velocity will be designated V_{LR} and is also calculated using the velocity addition formula. Note that the right frame sees our central observer frame as traveling to the left at velocity -*V* and our central reference frame sees the left reference frame traveling at velocity -*V*.

$$\frac{V_{LR}}{c} = \frac{-\frac{V}{c}-\frac{V}{c}}{1+\frac{V^2}{c^2}} = \frac{-\frac{2V}{c}}{1+\frac{V^2}{c^2}} \tag{6a}$$

$$\frac{v_R}{c} = \frac{\frac{v_L}{c}+\frac{V_{LR}}{c}}{1+\frac{v_L V_{LR}}{c^2}} \tag{6b}$$

After the collision, the total mass of the elements in Figure 5 is $m + M_2 + E_2/c^2$. The velocity of this mass relative to the right frame is V_{2R}. The unknown quantities can be found the Law of Conservation of Momentum and Law of Conservation of Energy.

$$\frac{mv_R}{\sqrt{1-\frac{v_R^2}{c^2}}} = \frac{\left(m + M_2 + E_2/c^2\right)V_{2R}}{\sqrt{1-\frac{V_{2R}^2}{c^2}}}$$

$$\frac{mc^2}{\sqrt{1-\frac{v_R^2}{c^2}}} + M_2c^2 = \frac{\left(m + M_2 + E_2/c^2\right)c^2}{\sqrt{1-\frac{V_{2R}^2}{c^2}}}$$

$$V_{2R} = \frac{v_R}{1+\left(\frac{M_2}{m}\right)\sqrt{1-\frac{v_R^2}{c^2}}} \tag{7a}$$

$$E_2 = mc^2\left(\frac{v_R\sqrt{1-\frac{V_{2R}^2}{c^2}}}{V_{2R}\sqrt{1-\frac{v_R^2}{c^2}}} - 1\right) - M_2c^2 \tag{7b}$$

The final momentum of the experiment can be found by first calculating the velocities

V_{1L} and V_{2R} relative to the central reference frame, where they will be V_1 and V_2.

$$V_1 = \frac{-V + V_{1L}}{1 - \frac{V_{1L}V}{c^2}} \qquad V_2 = \frac{V + V_{2R}}{1 + \frac{v_{2R}V}{c^2}} \tag{8a}$$

$$Momentum = 0 = \frac{(m + M_2 + E_2/c^2)V_2}{\sqrt{1 - \frac{V_2^2}{c^2}}} + \frac{M_1 V_1}{\sqrt{1 - \frac{V_1^2}{c^2}}} \tag{8b}$$

Verification of (8) is accomplished by putting equations (4) through (8) into a computer program and verifying that the system momentum is zero for any combination of variables.

Moving Energy Experiment

Figure 6 shows the second thought experiment.

Figure 6. Starting position of Moving Energy experiment.

In Figure 6, two masses are moving with the velocities shown relative to our stationary inertial reference frame. The left mass M_1 is attached to a smaller mass *m*.

$$M_2 = M_1 + m \tag{9}$$

Under the condition (9), these two masses will have equal momentums relative to our observing reference frame (though these momentums are opposite in direction).

After the experiment starts, mass *m* is converted into light energy. To simplify the calculations, it will be assumed that this light energy is a single photon which travels to the right toward mass M_2.

The first part of the experiment will be analyzed from a reference frame that is traveling to the left at speed *V*. See Figure 7, showing the photon with it's energy diminished by the amount of the M_1 kinetic energy.

Figure 7. Left reference frame view after mass *m* has been converted into a photon.

The energy of the photon is:

$$E_L = mc^2 - M_1 c^2 \left(\frac{1}{\sqrt{1 - \frac{V_{1L}^2}{c^2}}} - 1 \right) \tag{10}$$

The momentum of the photon is $P_L = E_L / c$ and the Law of Conservation of Momentum gives:

$$\frac{M_1 V_{1L}}{\sqrt{1 - \frac{V_{1L}^2}{c^2}}} = mc - M_1 c \left(\frac{1}{\sqrt{1 - \frac{V_{1L}^2}{c^2}}} - 1 \right)$$

$$\frac{V_{1L}}{c} = - \frac{\left(\frac{m}{M_1} + 1 \right)^2 - 1}{\left(\frac{m}{M_1} + 1 \right)^2 + 1} \tag{11}$$

In (11), V_{1L} has been made negative to indicate its direction. To observe the event where the photon runs into mass M_2, a reference frame traveling at speed *V* to the right is chosen. See Figure 8. The photon is still traveling with speed *c* in this reference frame. However, the energy of the photon is now diminished by the shifting of it's frequency as seen by the right reference frame (see (73) in the article *Energy Dynamics*). Note: it is assumed there are no gravitational effects exerted on the photon frequency in this experiment.

Figure 8. Right reference frame view of photon before colliding with M_2.

The frequency shift of the photon as observed by the right reference frame is a function of the observed velocity difference between the two frames. For this experiment, that velocity difference is V_{LR}, as given by (6a). Energy of the photon as seen by the right frame is:

$$E_R = \sqrt{\frac{1 - V_{LR}/c}{1 + V_{LR}/c}} \left[mc^2 - M_1 c^2 \left(\frac{1}{\sqrt{1 - \frac{V_{1L}^2}{c^2}}} - 1 \right) \right] \tag{12}$$

In (12), the absolute value of V_{LR} is used. After the photon has impacted M_2, it will have energy E_{RM}. This energy will add to the mass of M_2 and contribute to the final momentum of M_2 as seen by the right frame. If the velocity of M_2 after the collision is V_{2R}, then the Law of Conservation of Momentum gives:

$$\frac{E_R}{c} = \frac{\left(M_2 + E_{RM}/c^2\right)V_{2R}}{\sqrt{1 - \frac{V_{2R}^2}{c^2}}} \tag{13}$$

The Law of Conservation of Energy for the event in the right frame is:

$$E_R + M_2 c^2 = \frac{\left(M_2 + E_{RM}/c^2\right)c^2}{\sqrt{1 - \frac{V_{2R}^2}{c^2}}} \tag{14}$$

Which gives:

$$V_{R2} = \frac{c}{1 + \frac{M_2 c^2}{E_R}} \tag{15a}$$

$$M_2 + E_{RM} / c^2 = \frac{E_R \sqrt{1 - \frac{V_{2R}^2}{c^2}}}{c V_{2R}} \tag{15b}$$

The final momentum of the experiment can be found by first calculating the velocities V_{1L} and V_{2R} relative to the central reference frame, where they will be V_1 and V_2. This result is given by (8a). The final momentum of the experiment is:

$$Momentum = 0 = \frac{(M_2 + E_{RM} / c^2)V_2}{\sqrt{1 - \frac{V_2^2}{c^2}}} + \frac{M_1 V_1}{\sqrt{1 - \frac{V_1^2}{c^2}}} \tag{16}$$

Verification of (16) is accomplished by putting equations (9) through (16) into a computer program and verifying that the system momentum is zero for any combination of variables.

Conclusions

A solution technique has been presented for evaluation of momentum experiments in Special Relativity. Two experiments have been shown to have conserved momentum without a paradox. This does not change the paradox in the experiment of the article *A Momentum Paradox* because momentum was initially assumed to be conserved in that experiment. That was the paradox.

There was no paradox in the two experiments described in this article because the mechanism of momentum transfer between masses M_1 and M_2 was clearly defined and tools to evaluate this momentum transfer were already available in Special Relativity. The paradox of the device in the article *A Momentum Paradox* results from the lack of tools to evaluate the forces on it's components and resulting energy transfer through the device. The original stipulation that Special Relativity cannot include force or acceleration does not allow the analysis of complex devices. The assumption that General Relativity is the mechanism for understanding acceleration (and indirectly, force) is not satisfactory. General Relativity could not evaluate any of these mechanisms.

The experiments presented here also provide some support for the ideas presented in the articles *Moving Energy Forces* and *Energy Dynamics.* Those articles, and this article, also provide some peripheral support to the ideas presented in the article *Force and Energy Transfer Within Materials.*

This article also provides some further examples to illustrate how the choice of reference frame affects the view of the experiment. This is examined in the articles *Force and Geometry* and *Force and Time. Force and Geometry* shows that orthogonal coordinate axes are not always independent. *Force and Time* shows that different mechanisms providing the same force can produce different reactions in the time dimension. In this article, V_{LR} as given by (6a) shows that different reference frames must be careful in stating the directly observed velocities of each other. This occurs even though this case is the simplest one, with all frames moving coaxially. Special Relativity is defined by it's reference frames and their relative velocities. Being certain that all frames agree how they relate to each other is critical. Geometry (including time) in Special Relativity is not straightforward, as it is in Newtonian calculations.

The Law of Conservation of Momentum cannot be proved outright. Individual experiments can be evaluated to see if momentum is conserved. Then, if all these experiments show conservation, and no experiments show non-conservation, the Law is validated. The available information seems to suggest that the Law of Conservation of Momentum is observed within Special Relativity, but has conservation of momentum within Special Relativity been fully evaluated?