A simple formula for the force between magnetic charges is Gilbert's Model:

$$F = \frac{Uq_1q_2}{R^2} \qquad (1)$$

F - Magnetic force between two "charges"
U - A Permeability Constant = $\mu / 4\pi$
μ - permeability
q_1, q_2 - the two charges experiencing the force F
R - the distance between the two charges

The Uniform Linear Magnetic Field
The magnetic field of (1) is approximately spherically shaped around each of the charges. However, it will be useful to examine experiments using a magnetic field which is described by a straight line orthogonal coordinate system. This type of field is created using (1) and specifying that one of the charges is distributed along a rod of infinite length. This configuration is shown in Figure 1.

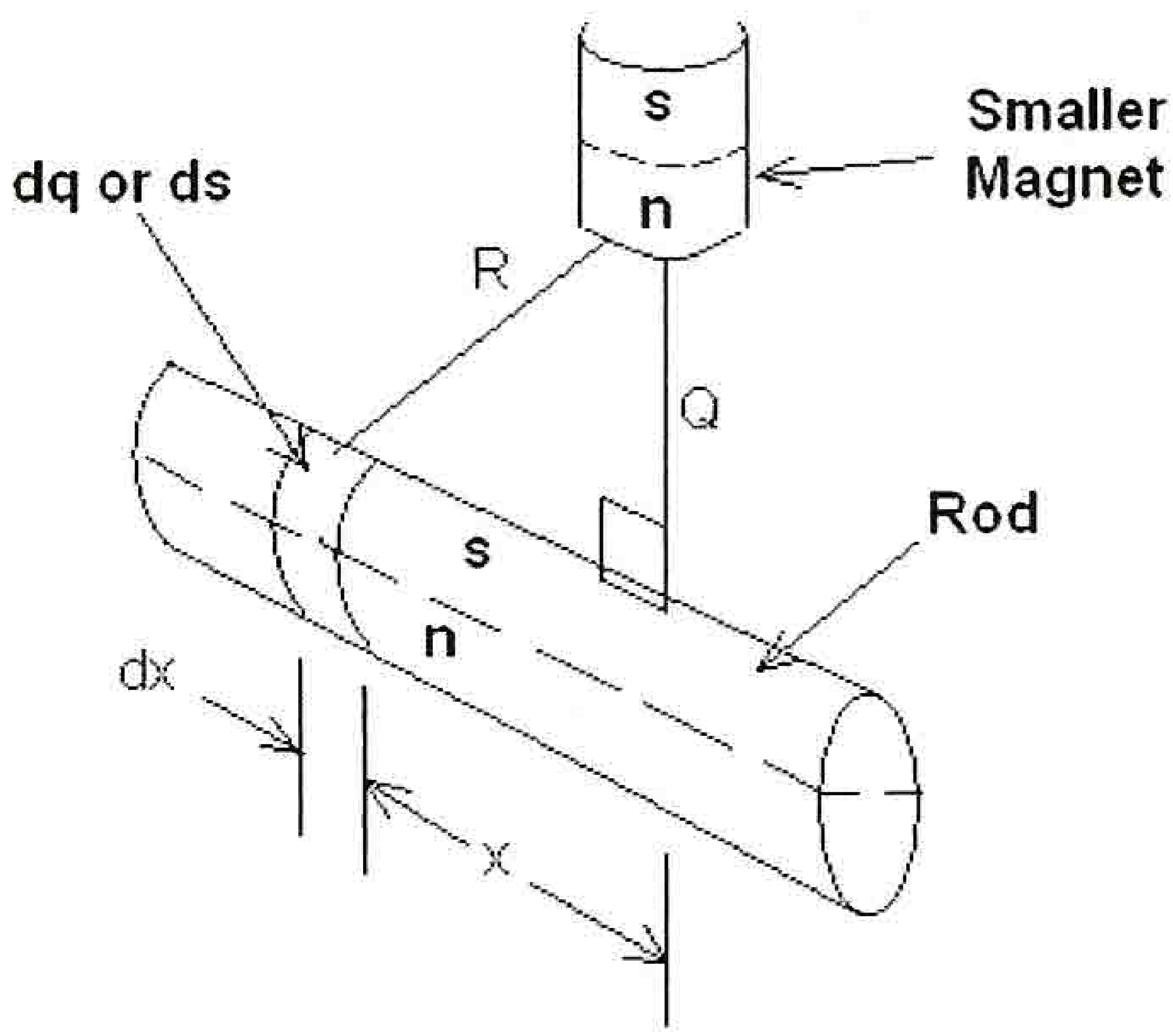

Figure 1. Infinitely long rod magnet creating a uniform linear magnetic field.

The magnets in Figure 1 have the charge distributions (poles) designated by the usual letters "n" and "s", with all the s charges on the top of the rod (near the smaller magnet) and all the n charges lined up along the bottom of the rod. The smaller magnet is arranged so that it is being attracted by the rod magnet, though the analysis could also apply for the opposite arrangement where the two magnets repel each other.

This arrangement can be analyzed using (1) for each of the four pairs of charge interactions (the n and s charges on the smaller magnet each being affected by the n and s charges on the rod magnet). To begin, just one case of these four will be derived and the other three cases are just variations of the first. The case to be examined is the n charge on the smaller magnet and the s charges on the rod magnet.

The magnetic field associated with this infinite rod is found by first specifying that charge s is an infinitesimal charge ds within the rod. The individual magnetic fields of all the ds charges will then be added up to give the total magnetic field of the rod.

The rod charge distribution is a uniform density of δ and will have a resulting infinitesimal charge ds = δ dx. Infinitesimal charge ds is located a distance x from the n charge of the smaller magnet. Charge n is located a distance Q above the rod. Using (1), the resulting magnetic force dF is:

$$dF = \frac{Un\,\delta ds x}{Q^2 + x^2} \qquad (2)$$

When viewed in the Q-x plane, the force dF can be seen to contribute to the force df (in the Q direction) between charge n and the rod. This is shown in Figure 2.

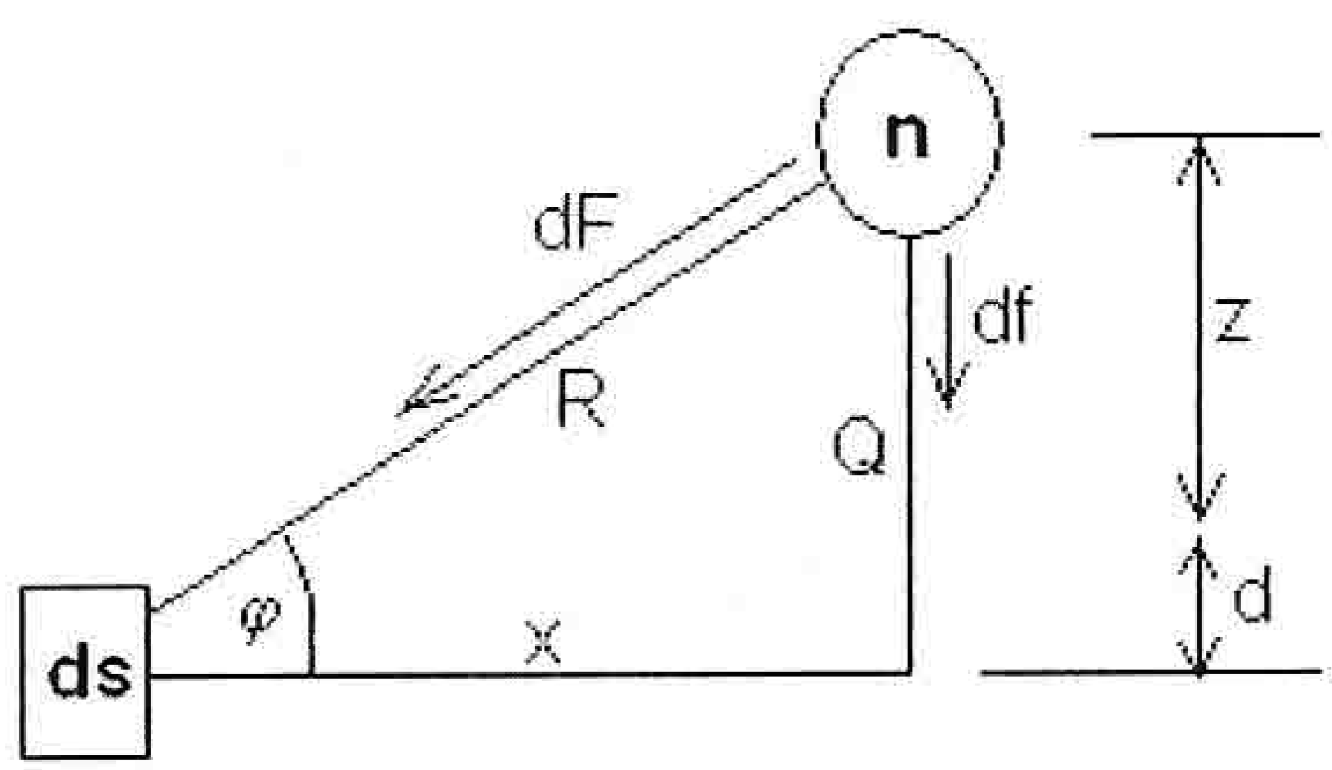

Figure 2. Q-x plane view of the forces.

The resulting equation for the force between charge n and the pole is:

$$df = dF \sin\phi = dF\left(\frac{Q}{\sqrt{Q^2 + x^2}}\right)$$

$$f = 2Un\,\delta Q \int_0^\infty \frac{dx}{\left(Q^2 + x^2\right)^{3/2}}$$

$$f = \frac{2Un\delta}{Q} = \frac{2Un\delta}{z+d} \qquad (3)$$

The distance Q is divided into two parts, z and d, defined in Figure 2. If a constant j is defined to be the magnetic acceleration divided by c, then the acceleration in the z direction can be defined as:

$$\frac{f}{2Un\delta} = \frac{\frac{1}{d}}{\frac{z}{d}+1} = \frac{j_z}{c}$$ for any z

$$\frac{j_0}{c} = \frac{1}{d}$$ for the case z = 0

$$j_z = \frac{j_0}{1 + \frac{j_0 z}{c}} \qquad (4)$$

The value of *j* can also be positive or negative, with positive indicating an attractive force and negative indicating a repellant force. Equation (4) is a magnetic acceleration that is in the same form as the dynamic acceleration (21) in the article *The Acceleration Law* and the gravitational acceleration (4) in the article *Special Relativity and Gravity.*

This special magnetic field will be referred to as a Dynamically Equivalent Orthogonal Magnetic Field (DEOMF). It is orthogonal because the magnetic force exists uniformly along the x-axis in the z direction.

Further Definition of the Experimental System

The configuration of Figure 1 can be altered so that experiments can be simplified. The magnetic dipole can be made approximately equal to the simpler gravitational field structure. See Figure 3.

In Figure 3, the infinitely long rod magnet has been replaced by and infinitely long plate magnet. In this configuration, the n and s charges are separated by a greater distance d3 than they were on the rod in Figure 1. A second plate magnet is offset a distance d4 from the first plate magnet. Also, its charges are reversed compared to the first plate magnet.

Two smaller magnets are lined up over top of the plate magnets in an arrangement that has both smaller magnets being attracted to both plate magnets. The total charges on the plate magnets are assumed much stronger than the charges on the smaller magnets. Distances d1 and d2 are responsible for the magnet force to the plate magnets. If d1 and d2 are significantly smaller than d3 and d4, then the experiment approximates a single smaller magnetic charge being attracted to a single plate (rod) charge distribution. A single magnetic charge calculation can reasonably describe the character of the magnetic field, much as a single calculation describes a gravity field.

The smaller magnets are shown lined up over top of one another, but this is just for clarity in the figure. There can be significant distance (x direction) between them, so that they don't interfere with each other. This may not be necessary if their magnetic charges are weak in comparison to the plate charges, in the same way that objects attracted to the earth are not assumed to gravitationally attract each other in a gravitational experiment.

This configuration will make it easier to understand the configuration of other experiments in this article.

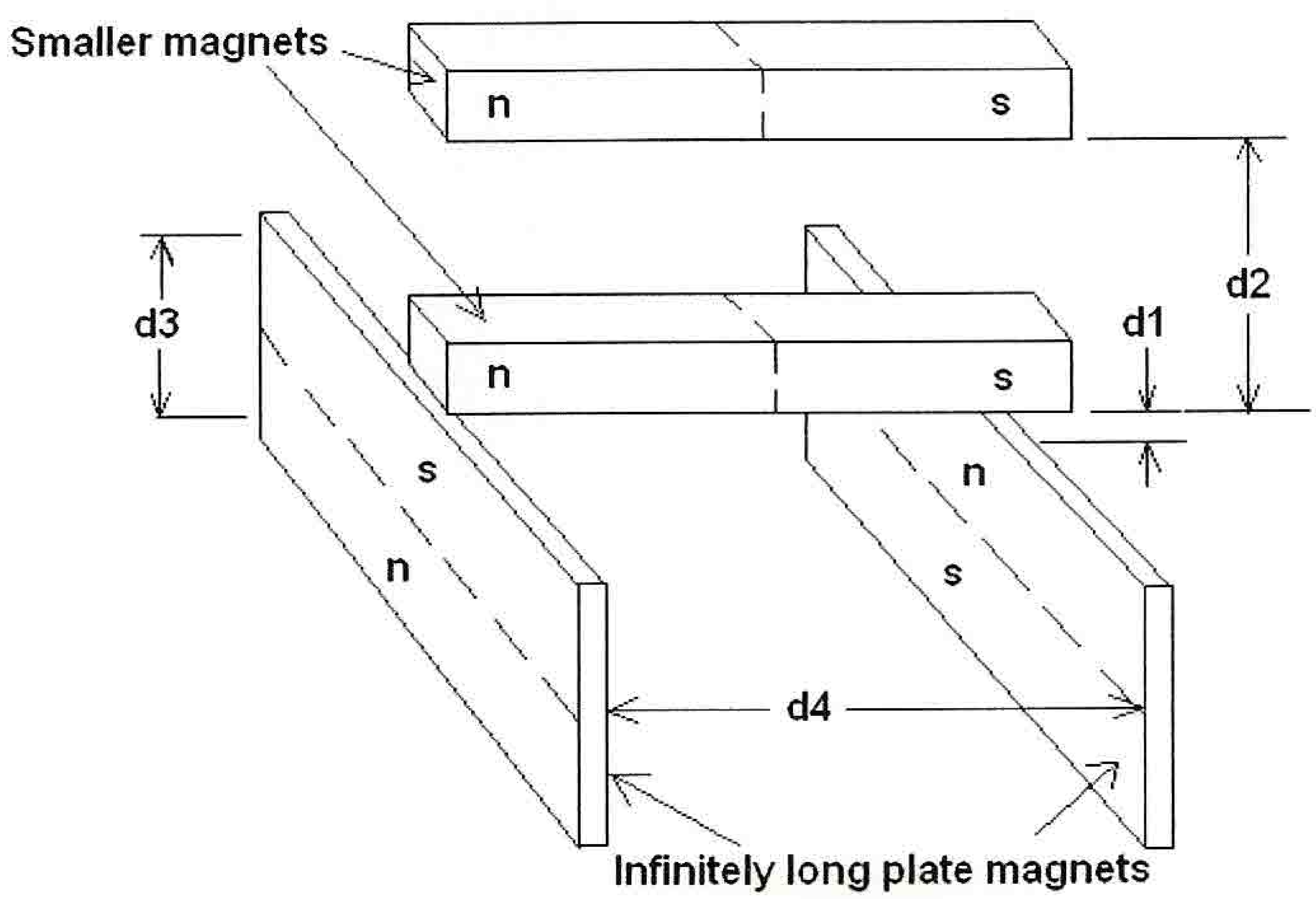

Figure 3. Alternate experimental configuration.

Force Transformation in a Magnetic Field

Consider a thought experiment where an object B has charge n and is stationary in reference frame B at a distance d above the rod magnet. An identical object A is stationary in reference frame A at a distance z above frame B. These two magnetic objects are far enough apart horizontally (in the x direction) so that they have an insignificant effect on each other in the experiment. Due to the different distances to the rod magnet, the magnetic force felt by someone holding the objects would be different. These forces are:

$F_B = mj_B c$ for object B $\quad$ $F_A = mj_A c$ for object A

$$F_A = F_B\left(\frac{j_A}{j_B}\right) = \frac{F_B}{1+\frac{j_B z}{c}} \tag{5}$$

Now assume that object A falls from frame A to frame B. The kinetic energy generated by this fall is KE. To find KE, note that object A becomes an inertial reference frame as soon as it starts to fall. Frame B accelerates towards object A with constant acceleration j_B. If the time interval that object A sees for the fall is t_A', then:

$$z = \frac{c}{j_B}\left[\left(1+(j_B t_A')^2\right)^{1/2} - 1\right]$$

$$j_B t_A' = \left[\left(1+\frac{zj_B}{c}\right)^2 - 1\right]^{1/2} \tag{6}$$

The velocity that object A has relative to frame B at the moment of impact is β_A and :

$$\beta_A = \frac{j_B t_A'}{\sqrt{1+(j_B t_A')^2}} \tag{7}$$

The kinetic energy of object A as it impacts frame B is:

$$KE = \frac{mc^2}{\sqrt{1-\beta_A^2}} - mc^2$$

$$\frac{1}{\sqrt{1-\beta_A^2}} = 1 + \frac{zj_B}{c}$$

$$KE = mczj_B \tag{8}$$

Now lets assume object A does not fall from frame A. Instead, it is lowered by observer B with a pole to frame B. Observer B does not know what force he will feel on his end of the pole. He therefore assumes this force will vary with z and calls it F(z). The work gained by observer B during this task will be W_{AB}.

$$W_{AB} = \int_{z}^{0} F(z)dz \tag{9a}$$

$$W_{AB} = F_{ave}z \tag{9b}$$

The value of F(z) would be applied for an incremental distance dz to give the expression for work in (9a). Another way to calculate the work would be to take the average force as the object was lowered through *z*, F_{ave}, and multiply it by *z*, as is shown in (9b). But, knowing $KE = W_{AB}$ gives:

$$F_{ave} = mcj_B = F_B \tag{10}$$

Equation (10) is true no matter what the value of z is. For any value of z, observer B always feels average force F_B on his end of the pole. Therefore, observer B always feels constant force F_B on his end of the pole at any *z*. This force will be called F_{BA}, and exists any time object A exerts force F_A on the opposite end of the pole.

$$F_{BA} = F_A\left(\frac{j_B}{j_A}\right) = F_A\left(1 + \frac{j_B z}{c}\right) = F_B \tag{11}$$

Horizontal Force in a Magnetic Field

In Figure 4, observer B pushes on a long board with a hole in it so that it passes around the rod magnet (plate magnet). The same board is part of an identical experiment on the opposite side of the rod magnet (not shown), which is positioned to eliminate any rotational movement or torque effects in the experiment. Horizontal force F_B is applied by observer B and horizontal force F_A is felt by observer A in frame A. The spring in frame A is compressed, clamped in the compressed position and lowered to frame B by observer B using a pole. The movement of the board in the x-direction is assumed to be measured equally by frame A and frame B.

$$x_A = x_B \tag{12}$$

When the board compresses the spring, a potential energy *E* is stored in the spring while in frame A. When the spring has been moved to frame B, the same energy and spring force must be present there. However, the work gained by lowering the spring down to frame B is zero. Energy (including light) is not affected by magnetic fields.

$$W_{AB} = 0 \tag{13}$$

The energy initially expended by observer B on the board is $\frac{1}{2}F_B dx$ and the energy stored in the spring is $E = \frac{1}{2}F_A dx$. When the spring is at frame B, the Law of Conservation of Energy gives:

$$\frac{1}{2}F_B dx = E + W_{AB} = \frac{1}{2}F_A dx + 0$$

$$F_A = F_B \tag{14}$$

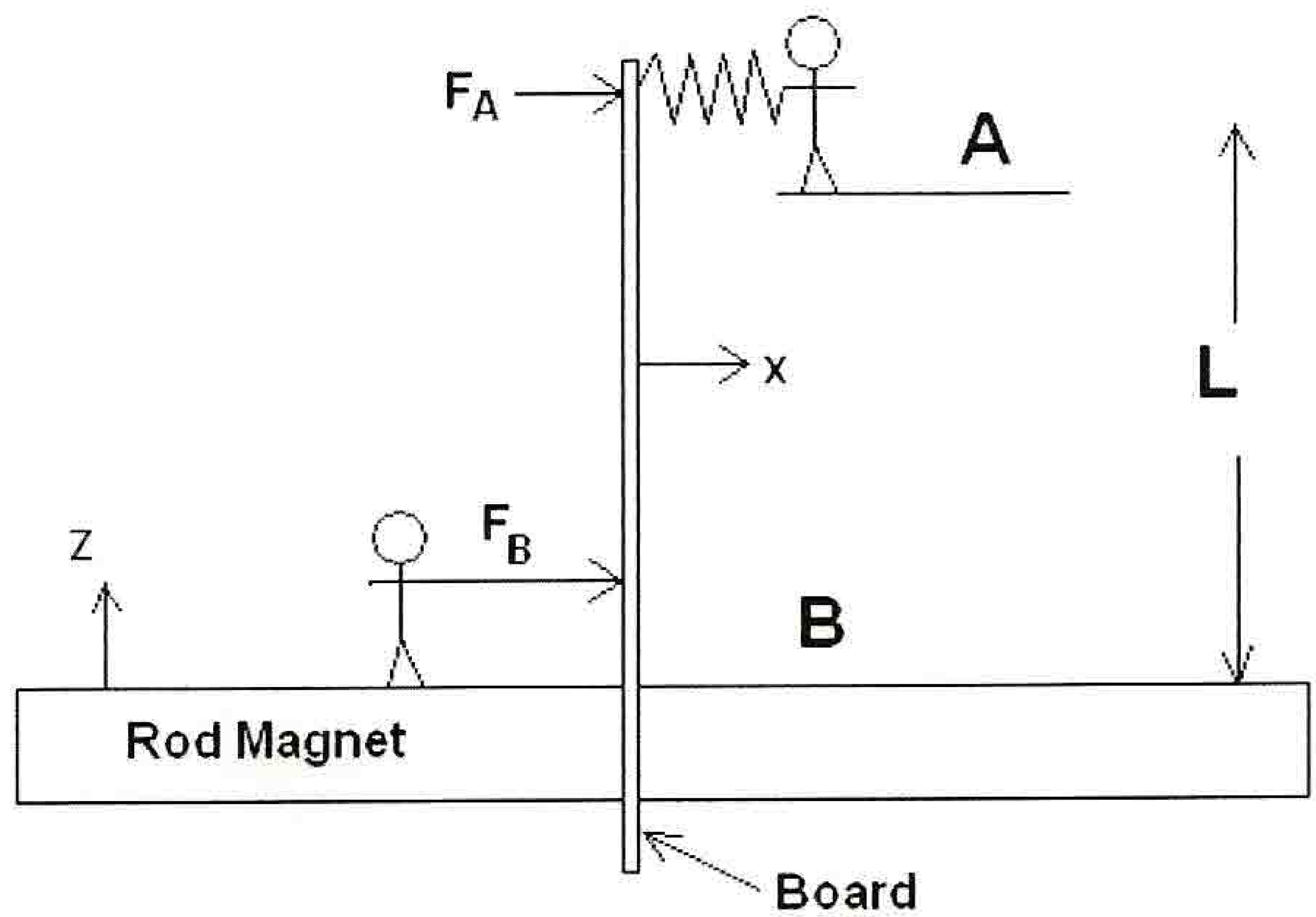

Figure 4. Horizontal force experiment.

The transformation (14) is a different result than was derived for gravitational fields. Magnets are selective in the objects that they influence. Gravity is not selective. The link between gravity and dynamic acceleration is perfect. Magnetism has only a partial similarity to dynamic or gravitational acceleration.

Time in a Magnetic Field

The transformation for time flow rates at different magnetic potentials is found from the experiment shown in Figure 5, where a board once again moves horizontally over the rod magnet. However, this time the frame B observer applies a force F_B to the board and simultaneously to a second mass in frame B going in the opposite direction. The mass in frame A will acquire a velocity to the right and the mass in frame B will acquired a velocity to the left. Both of these masses will be assumed to have magnetic charges that are attracted to the rod magnet (plate magnet). The experiment starts with all clocks reading zero and the total system momentum being zero.

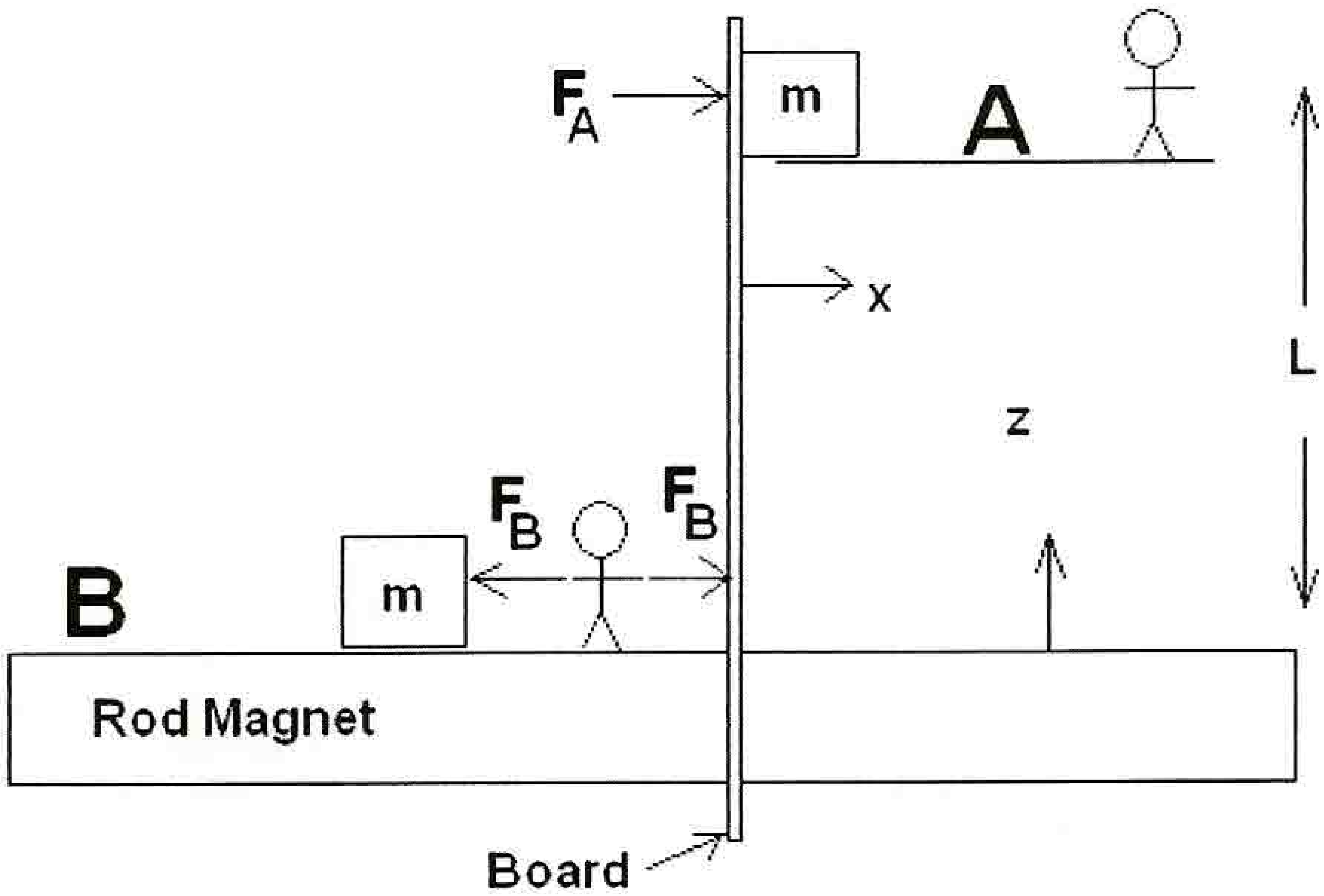

Figure 5. Time experiment.

Observer B applies the forces until clock reading t_B and the frame A observer sees the force at his location applied until clock reading t_A. The momentum produced in frame B must be equal in magnitude (and opposite in direction) to the momentum produced in frame A.

$$F_A t_A = F_B t_B$$

$$t_A = t_B \tag{15}$$

The Law of Conservation of Momentum links the force transformation in an acceleration to the time transformation. Therefore, the time transformation (15) is not the same as that for other dynamic or gravitational accelerations.

Verification of Time and Force Transformations

Transformations (14) and (15) are unique when compared to other accelerations. Will this inconsistency present problems (gravitational and dynamic accelerations have already been evaluated)? The Law of Conservation of Momentum will be calculated for the experiment of Figure 5 as a check on the validity of these new transformations. After the force application stops and the masses are both traveling with a constant velocity, the frame A mass (charge) will be placed in a box and the box will be "dropped" to frame B. See Figure 6.

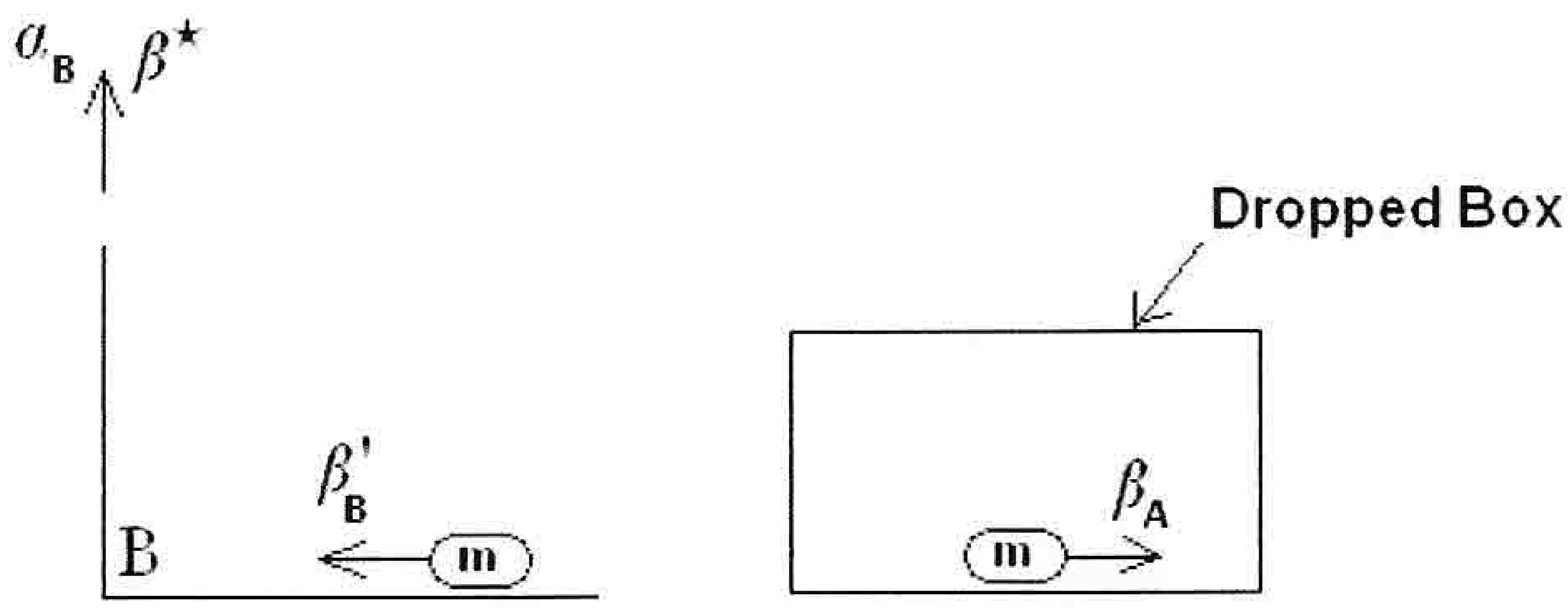

Figure 6. Law of Conservation of Momentum experiment.

At the instant that the box was "dropped" from frame A, it becomes an inertial reference frame and frame B accelerates up to it with constant acceleration j_B. At the instant shown in Figure 6, the two masses are at the same vertical coordinate (frame B). The box has no x-direction velocity and mass m is still traveling at its original velocity β_A. Frame B has accelerated to velocity β^*. From the reference frame of the box, the momentum of the mass inside the box is:

$$P_A = \frac{mc\beta_A}{\sqrt{1-\beta_A^2}} \qquad (16)$$

The velocity of the mass in frame B must now be calculated relative to the box. This will be done using (14) from the article *2 Dim. Position, Velocity, Acceleration* from the series of articles on Force and Space-Time. The reference frame labels in that article are backwards from the labels used in Figure 6, so care must be taken to keep track of the terms in this calculation. Quantities of Figure 6 will be shown in square parentheses and quantities of (14) will be shown without parentheses.

$\beta_X = [0]$ $\quad\beta_Y = [\beta^*]$ $\quad\beta_T = [\beta^*]$ Accelerating frame B velocities

$\beta_{XA} = [-\beta'_B]$ $\beta_{YA} = [0]$ Velocities of mass in frame B

$$K_S = 1 - \sqrt{1 - [\beta^*]^2} \qquad K_{TA} = \sqrt{1 - [\beta^*]^2}$$

$$\beta_{XB} = [\beta'_B]\sqrt{1 - [\beta^*]^2} \qquad \beta_{YB} = [\beta^*] \tag{17}$$

The velocity components of the accelerating frame B mass relative to the inertial box reference frame are β_{XB} and β_{YB} as shown in (17). The x-direction momentum of the frame B mass relative to the inertial box reference frame is P_{XB} and:

$$P_{XB} = \frac{mc\beta_{XB}}{\sqrt{1 - \beta_{XB}^2 - \beta_{YB}^2}} = \frac{mc[\beta'_B]\sqrt{1 - [\beta^*]^2}}{\sqrt{1 - [\beta'^2_B]}\sqrt{1 - [\beta^*]^2}} \tag{18}$$

From (14) and (15), it is already known that $\beta_A = \beta'_B$. Plugging this into (18) gives:

$$P_A = P_{XB} \tag{19}$$

In other words, the transformations (4), (5), (11), (12), (14) and (15) are compatible with the Law of Conservation of Momentum, even though some of these transformations differ significantly from those in the discussion surrounding Figure 15 in the article *Acceleration Dynamics*. Although the box sees the relativistic mass of the magnet in frame B as being greater than the relativistic mass of the magnet within the box, the slower time flow rate observed for frame B exactly compensates for the increase in mass as far as the Law of Conservation of Momentum is concerned.

Summary

A Dynamically Equivalent Orthogonal Magnetic Field can be constructed for magnetic fields in the same way as a DEOGF was done for a gravitational fields. Distance and field-aligned force transformations are identical to those for a gravitational field or a dynamic acceleration, but time and perpendicular-to-field force transformations are not. These relativistic differences between gravity and magnetism are caused by the absence of a magnetic effect on energy.